DECISION SUPPORT SYSTEM FOR RESERVOIR OPERATION USING ANALYTICAL MODELING RESULTS

BY

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THESIS

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ABSTRACT

This study applies an analytical approach to solving reservoir operation problems, in particular, developing an algorithm to search for the optimal solution for a system of reservoirs in parallel, establishing procedures to determine the effective forecast horizon, and demonstrating the practical applications of the derived rules. Based on these, a decision support system for reservoir operation is developed. More specifically, the analytical work of this thesis includes two parts. In the first part, a multi-stage optimization model is set up to derive the properties of optimal release decisions for a system of reservoirs in parallel with a single demand site; following that an algorithm is developed using the analytical results. In the second part, the properties of the optimal solution for a single water supply reservoir under uncertain forecast are derived, and these properties are then used to develop criteria and procedures to determine the effective forecast horizon, which can inform reservoir managers in the actual use of inflow forecast. Finally, a prototype reservoir operation decision support system is developed based on the analytical results. This system is to illustrate the applications of analytically derived reservoir operation rules to guide real-world reservoir operations. Through the system, users can use a graphical user interface (GUI) to upload data, execute model, and visualize the results. As a conclusion, being different from most existing studies using numerical models, this thesis shows the capabilities of the analytical approaches in providing information for real-world reservoir operation problems.

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TABLE OF CONTENTS

CHAPTER 1: INTRODUCTION

Reservoir operation is a fundamental issue for water resources planning and management. Extensive efforts have been made to develop effective reservoir operation policies including simulation and optimization models (Yeh, 1985; Wurbs,1993; Labadie, 2004). While the majority of the research has been with numerical approaches, such as classic mathematical programming models, heuristic methods, and dynamic programming techniques, which rely on computer programs to solve for solutions (e.g., Stedinger, Sule, & Loucks, 1984; Wardlaw & Sharif, 1999), some studies used analytical approach to derive general rules and insights for reservoir operation rules (e.g., Lund & Guzman,1999; You & Cai, 2008a, 2008b). The most remarkable reservoir operation rule obtained from analytical derivations is probably the NYC rules (Clark, 1950) for parallel reservoir system operation, which minimizes the total expected spills from all the reservoirs in the system, and the NYC rules have been used for a long time.

Instead of minimizing spills or maximizing total available water though a numerical model, Draper & Lund (2004) introduced concave economic benefit functions for both the release and the carry-over storage, and set up a general economic principle to obtain the optimal operation policy to maximize the total economic benefit of the current release and the carry-over storage for the future. You & Cai (2008a) extended this work by explicitly including uncertainty in future inflow and developed hedging policies with a conceptual two-stage model. Following that, the economic views and analytical optimization approaches are further applied to addressing the various issues of reservoir operation. Zhao et al. (2011) formulated a two-stage model to maximize the total benefit of the releases over two stages, an optimal solution was solved using Karush–Kuhn–Tucker KKT conditions, and an algorithm was developed based on the analytical results to solve the problem numerically. Following Zhao et al. (2011), Xu et al. (2014) extended the two-stage model to a multi-stage model with explicitly incorporated uncertain inflow forecast, and developed a numerical algorithm based on the optimality conditions. Instead of maximizing the total economic benefits, Shiau (2011) derived a two-point hedging rule for a single reservoir operation by minimizing a loss function in terms of the deviations of the current release and carry-over storage from the predetermined targets. Zeng et al. (2015) extended the analysis from a single reservoir to a system of multi-reservoirs in parallel, and derived optimal operation policies by setting the loss function in terms of the deviations of both the individual reservoir carry-over storages and the system-level release decisions from the predetermined targets. Studies are also conducted for flood management purposes, including studies on hedging between post-flood water conservation and flood risk (e.g., Ding et al., 2015, 2017; Wan et al., 2016) and studies on pre-releasing to allocate storage capacity between floods at current stage and future stages (e.g., Zhao et al., 2014; Hui & Lund, 2015).

This work follows the analytical frame work by Draper & Lund (2004), You & Cai (2008a) to explore more capabilities of the analytical approaches in providing solutions for reservoir operation related issues, e.g., algorithms development, theory proof, tool design, etc.

Due to the system complexity, deriving operation policies for a system of reservoirs in parallel is challenging (Oliverira and Loucks, 1997; Labadie, 2004). Most research relies on numerical methods, especially, heuristic methods, to explore operation policies for multi-reservoir systems (e.g., Chang & Change, 2009; Jalali et al., 2007; Wardlaw & Sharif, 1999). On the one hand, though reasonable results might be obtained from these numerical methods, the general properties of optimal operation policies for parallel reservoir systems can hardly be represented, explicitly. On the other hand, many of these numerical methods, especially the widely used stochastic dynamic programming approach, suffer from computational burden induced by the high dimensionality of the problems (Castelletti et al., 2010; Jairaj & Vedula, 2003). As compared to the most widely-used numerical approaches, analytical approaches, though limited by assumptions and simplifications, provide optimal solutions for a system of reservoirs in parallel with less computational requirements and more general insights. Zeng et al. (2015) derived the optimal operation policy for a multi-reservoir system in parallel with a joint demand site by minimizing a loss function in terms of the deviation of carry-over storage of the individual reservoirs and the system-level release decisions from the predetermined targets. However, in Zeng et al. (2015), the coordination of the operation of reservoirs in the system can only be partially revealed as the coordination also depends on the predetermined targets. To explore general insights for coordinating the operation of a system of reservoirs in parallel and develop computationally efficient algorithms, this thesis will analytically derive the operation rules for such a system with a single demand site by maximizing a benefit function evaluated at the demand site through a multistage optimization model; following that, an algorithm will be developed to solve the optimization problem. Being different from Zhao et al. (2011) and Xu et al. (2014), who designed algorithms for single reservoir operation by recursively searching the solution based on the optimality conditions, in this study, the algorithm will be designed based on the insights obtained from the analytical results, and will solve the problem efficiently despite for the complexity with the high-dimensional system.

The concept of hedging in reservoir operation was first provided by Bower et al. (1962), and has been widely explored (Neelakantan & Sasireka, 2015). By hedging rules, water will be allocated between current and future stages. Instead of satisfying the water demand at current stage with the first priority, as the standard operation policy (SOP) proposes, hedging rules allow reducing release at the current stage and reserve some amount of water for the future to mitigate possible water stress in the future. Inflow forecast, thus, is of great importance to achieve better hedging performance. It has been demonstrated that inflow forecast is important for making reservoir operation decisions (Zhao et al., 2011), while imperfect forecast might significantly reduce the usefulness of the forecast information (Mishalani and Palmer, 1988). To address the issues of determining an appropriate forecast horizon for reservoir operation decision making, You and Cai (2008c) developed a theoretical relationship for determining the forecast horizon using dimensional analysis. The forecast horizon is defined as the length of the forecast beyond which the inflow will no longer affect the release decision at the current stage (e.g., in decision horizon.) Zhao et al. (2012) conducted numerical experiments with imperfect forecast and proposed the concept of effective forecast horizon (EFH) with a certain level of uncertainty, which provides maximum information to support reservoir operation decisions. Though promising concepts are provided by You and Cai (2008c) and Zhao et al. (2012), procedures are still needed to determine the EFH. In this thesis, a relationship between the forecast uncertainty and the decision uncertainty is derived analytically, which is further applied to providing criteria and procedures for determining the EFH.

Though there is growing amount of research applying analytical approaches to addressing reservoir operation issues, there exists rarely any decision support tool based on these results for real world reservoir operations. Most existing reservoir operation decision support tools rely on numerical methods for simulation or optimization (e.g., Koutsoyiannis et al., 2002), and a large portion of them are case-specific (e.g., Chandramouli & Deka, 2005). Thus, to provide a new approach and to show the potential of using analytically derived results, a prototype reservoir operation decision support system is developed based on the analytical results obtained from this and other studies.

The rest of thesis will be organized as follows. In the Chapter 2, a multi-stage optimization model is set up to discuss the properties of the optimal release decision for a system of multi-reservoir in parallel and an algorithm is further developed based on the analytical results; In the Chapter 3, a multi-stage reservoir operation model is set up for a single reservoir with a single demand site; following that, criteria and procedures for determining the EFH are developed. In Chapter 4, a prototype reservoir operation decision support system is described and demonstrated. Finally, conclusions are provided in Chapter 5.

CHAPTER 2: GENERAL OPERATION RULES FOR SYSTEM OF RESERVOIRS IN PARALLEL

2.1 Background

In Zeng et al. (2015), hedging rules for a system of reservoirs in parallel has been discussed with a two-stage model given a storage target for each reservoir and a system level release target, and a two-point hedging curve is derived. However, the work can only partially represent the actual cooperation between reservoirs in parallel since part of the actual cooperation may be reflected by the given storage and release targets. Following Zeng et al. (2015), we derive general rules for the operation of a system of reservoirs in parallel using a multi-stage model with the utility function evaluated at a single demand site, which attempts to include more real-world corporation rules among parallel reservoirs associated with a water supply system. Cooperation between parallel reservoirs is investigated, and an algorithm is developed based on the analytical results. The results from this section could reveal some insights for parallel reservoir operation and can also be used to provide optimal operation decisions for real-world operation of a system of reservoirs in parallel.

2.2 Model Formulation and Discussion

A parallel reservoir system with M parallel reservoirs supplying water for a single demand site is investigated, the water supply system is illustrated in Figure 2.1.

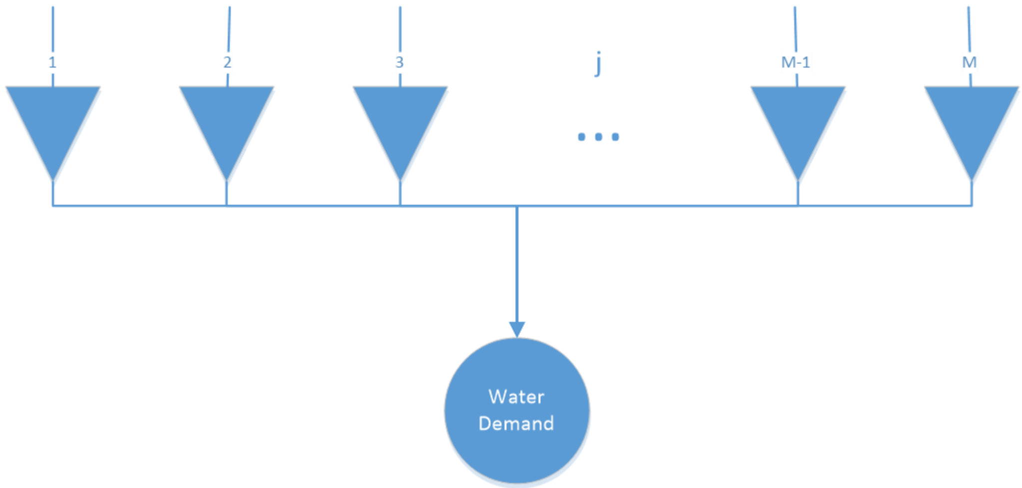


Figure 2.1 A system of reservoirs in parallel

Assume there are M parallel reservoirs, with perfect forecast of length N, the following multistage reservoir operation model could be set up.

$$\text{Obj. } \max \sum_{i=1}^{N} b_i\left(\sum_{j=1}^{M} r_{i,j}\right) \tag{2.1}$$

$$s.t.$$

$$s_{i-1,j} + i_{i,j} - r_{i,j} = s_{i,j}, \quad for\ i = 1, \dots, N\ and\ j = 1,2, \dots, M \tag{2.2}$$

$$s_{i,j} \geq 0, \quad for\ i = 1, \dots,\ N-1\ and\ j = 1,2, \dots, M \tag{2.3}$$

$$s_{i,j} \leq K_j, \quad for\ i = 1, \dots,\ N-1\ and\ j = 1,2, \dots, M \tag{2.4}$$

$$r_{i,j} \geq 0, \quad for\ i = 1,2, \dots,\ N\ and\ j = 1,2, \dots, M \tag{2.5}$$

Here, i is the index of time and j is the index of reservoirs, M is the total number of reservoirs and N is the total number of stages, $b_i(r)$ is the concave utility function of water supply at stage i, $s_{i,j}$ is the storage of reservoir j at the end of stage i, the initial and ending storage of reservoir j is given as $s_{0,j}$ and $s_{N,j}$, $r_{i,j}$ is the release of reservoir j at stage i, $i_{i,j}$ is the inflow to reservoir j during stage i, and K_j is the storage capacity of reservoir j.

This nonlinear programming model could be solved by KKT conditions (Bazaraa et al., 2013) given by Eq. (2.6) – Eq. (2.14).

$$-\nabla\sum_{i=1}^{N} b_i\left(\sum_{j=1}^{M} r_{i,j}\right)+\nabla\sum_{i=1}^{N}\sum_{j=1}^{M}\lambda_{b,i,j}\left(s_{i,j}-s_{i-1,j}-i_{i,j}+r_{i,j}\right)+\nabla\sum_{i=1}^{N-1}\sum_{j=1}^{M}\lambda_{e,i,j}\left(-s_{i,j}\right)$$

$$+\nabla\sum_{i=1}^{N-1}\sum_{j=1}^{M}\lambda_{f,i,j}\left(s_{i,j}-K_j\right)+\nabla\sum_{i=1}^{N}\sum_{j=1}^{M}\lambda_{r,i,j}\left(-r_{i,j}\right)\ ,$$

$$for\ i=1,\dots,N-1\ and\ j=1,2,\dots,M \tag{2.6}$$

$$-s_{i,j}\le 0,\qquad for\ i=1,\dots,N-1\ and\ j=1,2,\dots,M \tag{2.7}$$

$$s_{i,j}\le K_i,\qquad for\ i=1,\dots,N-1\ and\ j=1,2,\dots,M \tag{2.8}$$

$$\lambda_{e,i,j}s_{i,j}=0,\qquad for\ i=1,\dots,N-1\ and\ j=1,2,\dots,M \tag{2.9}$$

$$\lambda_{f,i,j}\left(s_{i,j}-K_i\right)=0,\qquad for\ i=1,\dots,N-1\ and\ j=1,2,\dots,M \tag{2.10}$$

$$\lambda_{r,i,j}r_{i,j}=0,\qquad for\ i=1,\dots,N\ and\ j=1,2,\dots,M \tag{2.11}$$

$$\lambda_{e,i,j}\ge 0,\qquad for\ i=1,\dots,N-1\ and\ j=1,2,\dots,M \tag{2.12}$$

$$\lambda_{f,i,j}\ge 0,\qquad for\ i=1,\dots,N-1\ and\ j=1,2,\dots,M \tag{2.13}$$

$$\lambda_{r,i,j}\ge 0\ \ for\ i=1,2,\dots,N\ and\ j=1,2,\dots,M \tag{2.14}$$

Here, $\lambda_{b,i,j}$ is the shadow price for the mass balance constrain, Eq. (2.2), of reservoir *j* at stage *i*, $\lambda_{e,i,j}$ is the shadow price for non-negative storage constraints, Eq. (2.3), of reservoir *j* at the end of stage *i*, $\lambda_{f,i,j}$ is the shadow price for storage capacity constraints, Eq. (2.4), of reservoir *j* at the end of stage *i*, and $\lambda_{r,i,j}$ is the shadow price for non-negative release constraints, Eq. (2.5), of reservoir *j* on stage *i*.

To simplify representation, let r_i represent the sum of release at stage *i* from all reservoirs.

$$r_i=\sum_{j=1}^{M} r_{i,j} \tag{2.15}$$

Note, at the same stage, the following formula should be satisfied according to the property of derivatives as shown in Eq. (2.16).

$$\frac{\partial b_i\left(\sum_{j=1}^{M} r_{i,j}\right)}{\partial r_i}=\frac{\partial b_i\left(\sum_{j=1}^{M} r_{i,j}\right)}{\partial r_{i,j}}=b_i'\left(\sum_{j=1}^{M} r_{i,j}\right) \tag{2.16}$$

Physically, this means that, at stage i, the marginal utility (MU) of release from each reservoir is identical to each other.

By KKT conditions, following relationship could be obtained (See Appendix A).

$$b'_{i+1}\left(\sum_{j=1}^{M} r_{i+1,j}\right) + \lambda_{e,i,j} + \lambda_{r,i+1,j} = b'_{i}\left(\sum_{j=1}^{M} r_{i,j}\right) + \lambda_{f,i,j} + \lambda_{r,i,j} \tag{2.17}$$

Three different scenarios are discussed based on the relationships between the marginal utilities (MU) at two consecutive stages (as shown in Figure 2.2).

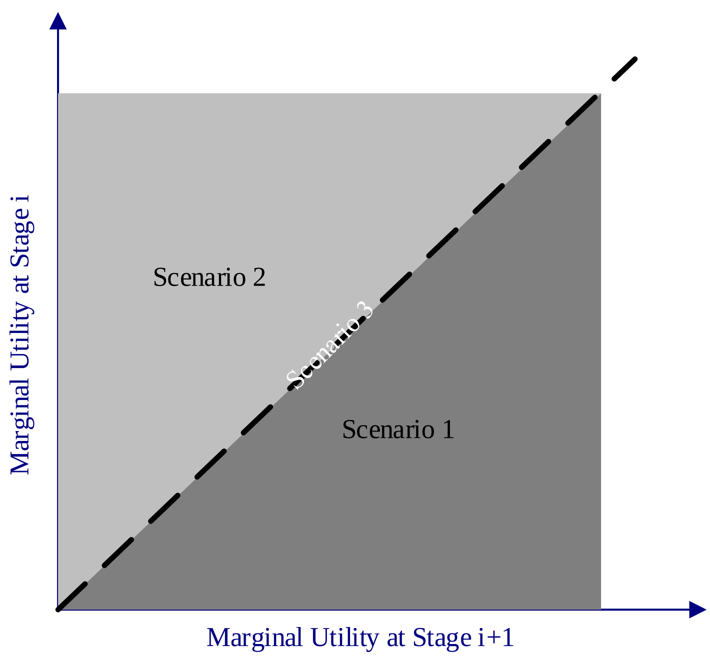


Figure 2.2 Illustration of three different scenarios

Scenario 1:

If for two consecutive stages (stage i and i+1), we have the following relationship, i.e., the MU level becomes higher at next stage,

$$b'_{i+1}\left(\sum_{j=1}^{M} r_{i+1,j}\right) > b'_{i}\left(\sum_{j=1}^{M} r_{i,j}\right) \tag{2.18}$$

then,

$$\lambda_{r,i,j} + \lambda_{f,i,j} = P + \lambda_{r,i+1,j} + \lambda_{e,i,j}\,, \qquad for\ j = 1,2,\ldots,M \tag{2.19}$$

where P is a positive number as defined below:

$$P = b'_{i+1}\left(\sum_{j=1}^{M} r_{i+1,j}\right) - b'_{i}\left(\sum_{j=1}^{M} r_{i,j}\right) > 0 \tag{2.20}$$

All reservoirs must satisfy the relationship shown in Eq. (2.19). Any of the reservoirs can have one of the following three cases:

(1) If $\lambda_{e,i,j}$=0 (i.e., reservoir *j* is usually not empty at the end of stage *i*) and $\lambda_{f,i,j} = 0$ (i.e., reservoir *j* is usually not full at the end of stage *i*),

$$\lambda_{r,i,j} = P + \lambda_{r,i+1,j} \tag{2.21}$$

Therefore,

$$\lambda_{r,i,j} > 0 \tag{2.22}$$

This means there is no release from the reservoir at stage *i* (by Eq. (2.22)). This shows that the following three conditions might match together as part of the optimization conditions: the MU level increases from stage *i* to stage *i*+1 (Eq. (2.18)), the storage is at any level[1] at the end of stage *i,* and the reservoir does not release at stage *i* (Figure 2.3(a)). Under this case, as the MU increases, the reservoir tends to save more water to the future stage with higher MU, and as a result, all inflow at stage *i* is kept in the reservoir. This indicates that the reservoir capacity is not the controlling factor and the inflow condition limits the ability for the reservoir to save more water to the future.

(2) If $\lambda_{e,i,j}$>0 (i.e., reservoir *j* is empty at stage at the end stage *i*), and we must have $\lambda_{f,i,j} = 0$ (i.e., the reservoir *j* is not full at the end of stage *i*), then:

$$\lambda_{r,i,j} = P + \lambda_{e,i,j} + \lambda_{r,i+1,j} \tag{2.23}$$

Therefore,

$$\lambda_{r,i,j} > 0 \tag{2.24}$$

[1] Under extreme cases, the reservoir can be full but no benefit associated with keeping more water in the reservoir or empty but no benefit associated with releasing more water from the reservoir.

This means there is no release from the reservoir at stage i (Eq. (2.24)). This shows that the following three conditions might match together as part of the optimization conditions: the MU level increases from stage i to stage i+1, the reservoir is empty at the end of stage i and the reservoir does not release at stage i (Figure 2.3(b)). This is only possible if there is no inflow during stage i and the initial storage of stage i is zero (otherwise there should be some water saved at the end of i). Under such conditions, the reservoir is dry and has no release. Even though the MU increases, the inflow condition limits the ability for the reservoir to save water to the future stages with higher MU.

(3) If $\lambda_{e,i,j}$=0 (i.e., reservoir j is not empty at the end of stage i), and $\lambda_{f,i,j} > 0$ (i.e., reservoir j at stage is full at the end of stage i),

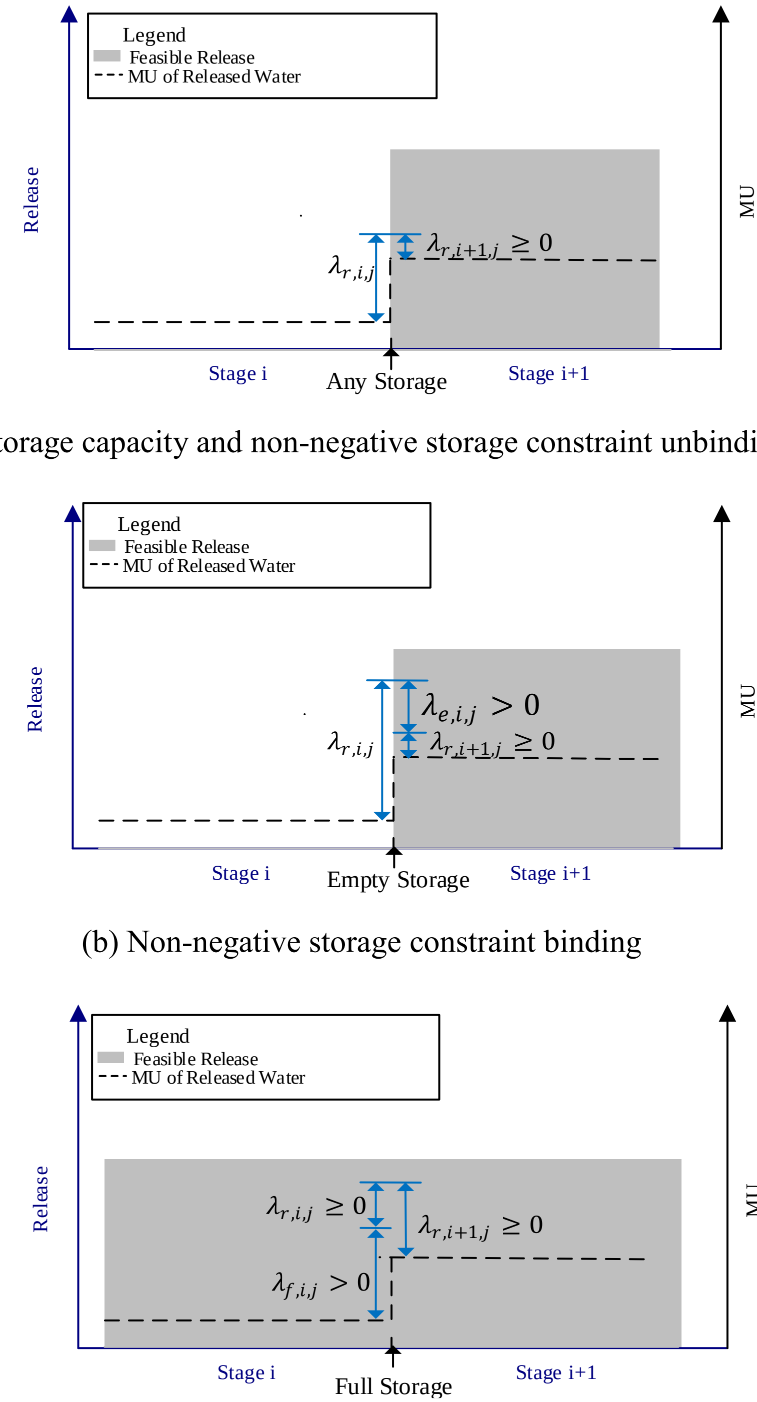
$$\lambda_{r,i,j} + \lambda_{f,i,j} = P + \lambda_{r,i+1,j} \tag{2.25}$$

Thus, either $\lambda_{r,i,j} > 0$ or $\lambda_{r,i,j} = 0$ is possible, and thus, the reservoir can release or not release during stage i. This shows that the following two conditions might match together as part of the optimization conditions: the MU level increases from stage i to stage i+1, the reservoir becomes full at the end of stage i (Figure 2.3(c)). Under this case, as the MU increases, the reservoir tends to save more water to the future stage with higher MU, and as a result, the reservoir is full at the end of stage i. This indicates that the reservoir capacity limits the ability for the reservoir to save more water to the future.

As a summary, under Scenario 1, when the MU level increases from stage i to i+1, one of the following situations must occur for any reservoir in the system under an optimal solution:

a. Having a full storage at the end of stage i (corresponds to case (3) with limited capacity);

b. Having no release during stage i and save water to stage i+1 (corresponds to case (1) and (2) with limited inflow).



(a) Storage capacity and non-negative storage constraint unbinding

(b) Non-negative storage constraint binding

(c) Storage capacity constraint binding

Figure 2.3 Different cases with increasing MU

Scenario 2:

If for two consecutive stages (stage i and i+1), we have the following relationship, i.e., the MU becomes lower on next stage (i+1),

$$b'_{i+1}\left(\sum_{j=1}^{M} r_{i+1,j}\right) < b'_{i}\left(\sum_{j=1}^{M} r_{i,j}\right) \tag{2.26}$$

then,

$$\lambda_{r,i,j} + \lambda_{f,i,j} + P = \lambda_{e,i,j} + \lambda_{r,i+1,j}\,, \qquad for\ j = 1,2,\dots,M \tag{2.27}$$

where P is a positive number with the following definition:

$$P = b'_{i}\left(\sum_{j=1}^{M} r_{i,j}\right) - b'_{i+1}\left(\sum_{j=1}^{M} r_{i+1,j}\right) > 0 \tag{2.28}$$

All reservoirs must satisfy the relationship shown in Eq. (2.27). Any of the reservoirs can have one of the following three situations:

(1) If $\lambda_{e,i,j}$=0 (i.e., reservoir j is usually not empty at the end of stage i) and $\lambda_{f,i,j} = 0$ (i.e., reservoir j is usually not full at the end of stage i),

$$\lambda_{r,i,j} + P = \lambda_{r,i+1,j} \tag{2.29}$$

Therefore,

$$\lambda_{r,i+1,j} > 0 \tag{2.30}$$

This means there is no release from the reservoir at stage i+1. This shows that the following three conditions might match together as part of the optimization conditions: the MU decreases from stage i to stage i+1, the storage could be at any level at the end of stage i[2] and the reservoir does not release at stage i+1 (Figure 2.4(a)). Under this case, as the MU is decreasing, the reservoir tends to use more water at the current stage with higher MU. However, the reservoir may not be empty at the end of stage i. One possible explanation is that, if the reservoir is large enough, it can store part of the inflow from the current stage

[2] Under some special cases, it is possible for the reservoir to be full but no benefit associated with keeping more water in the reservoir or empty but no benefit associated with releasing more water from the reservoir.

and all inflows in the coming stages with lower MU than that of the current stage, for a longer time to a future stage when the MU level is even higher than the current MU. This indicates that the situation that a relative large capacity and limited inflow make it possible for the reservoirs not to be empty and still store some water for the future when the MU decreases.

(2) If $\lambda_{e,i,j}$>0 (i.e., reservoir *j* is empty at stage at the end of stage *i*), and we must have $\lambda_{f,i,j} = 0$ (i.e., the reservoir *j* is not full at the end of stage *i*), then:

$$\lambda_{r,i,j} + P = \lambda_{r,i+1,j} + \lambda_{e,i,j} \tag{2.31}$$

Thus, the reservoir can release or not release on both stage *i* and stage *i*+1. This shows that the following two conditions might match together as part of the optimization conditions: the MU decreases from stage *i* to stage *i*+1 and the reservoir becomes empty at the end of stage *i*, (Figure 2.4(b)). Under this situation, as the MU is decreasing, the reservoir tends to use more water at the current stage; as a result, the reservoir becomes empty at the end of stage *i*. From another point of view, the reservoir is making space to store inflows during the coming stages with lower MU. Thus, the storage capacity is the actual limiting factor in this case.

(3) If $\lambda_{e,i,j}$=0 (i.e., reservoir j is not empty at the end of stage *i*), and $\lambda_{f,i,j} > 0$ (i.e., reservoir *j* at stage is full at the end of stage *i*),

$$\lambda_{r,i,j} + P + \lambda_{f,i,j} = \lambda_{r,i+1,j} \tag{2.32}$$

Therefore,

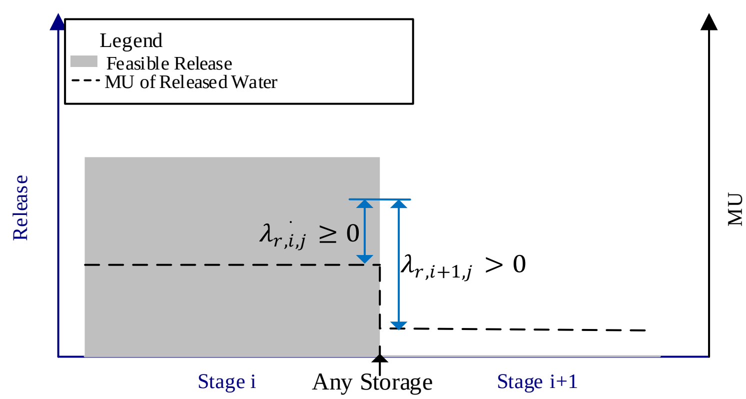
$$\lambda_{r,i+1,j} > 0 \tag{2.33}$$

This means there is no release from the reservoir at stage *i*+1. Thus, the following three conditions might match together as part of the optimization conditions: the MU level decreases from stage *i* to stage *i*+1, the reservoir becomes full at the end of stage *i*, and the reservoir has no release at stage *i*+1 (Figure 2.4 (c)). This is only possible if there is no inflow during stage *i*+1 and the ending storage of stage *i*+1 is full too. Under such conditions, the reservoir is full and has no release. Similar as Case (1) under Scenario 2, the limited inflow makes it possible for reservoirs not to become empty when the MU decreases.

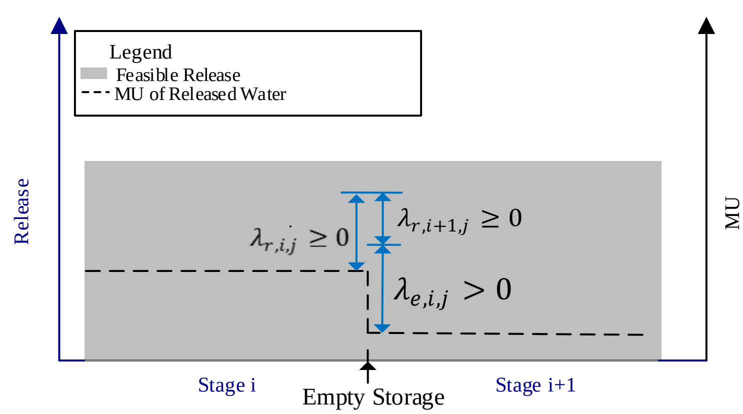
As a summary, under Scenario 2, when the MU level decreases from stage i to i+1, one of the following situations must occur for any reservoir in the system with the optimal solution:

a. having an empty storage at the end of stage i (corresponds to case (2) with a limited capacity);

b. having no release at stage i+1, i.e., having enough capacity to store all inflow during the coming stages with low MU (corresponds to case (1) & (3) with limited inflow).



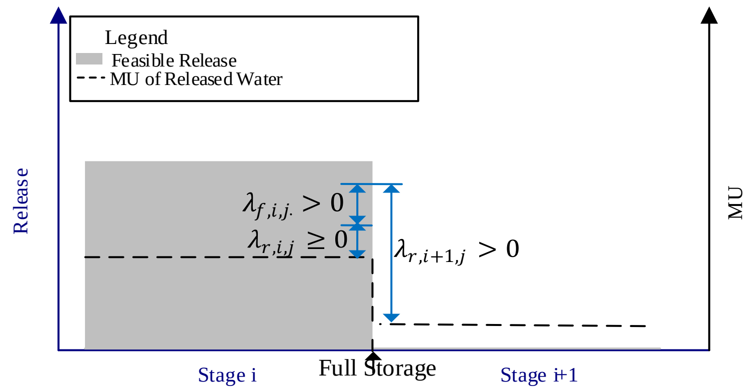
(a) Storage capacity and non-negative storage constraints unbinding



(b) Non-negative storage constraint binding

Figure 2.4 Different cases with decreasing MU

Figure 2.4 (cont.)



(c) Storage capacity constraint binding

Scenario 3:

In this scenario, storage and release conditions for two consecutive stages with identical MU are considered,

$$b'_{i+1}\left(\sum_{j=1}^{M} r_{i+1,j}\right) = b'_i\left(\sum_{j=1}^{M} r_{i,j}\right) \tag{2.34}$$

The following relationship is achieved between these two stages.

$$\lambda_{r,i,j} + \lambda_{f,i,j} = \lambda_{e,i,j} + \lambda_{r,i+1,j} \ , \qquad for\ j = 1,2,\ldots,M \tag{2.35}$$

All reservoirs must satisfy the relationship shown in Eq. (2.35). Any of the reservoirs can have one of the following three situations:

(1) If $\lambda_{e,i,j}$=0 (i.e., reservoir *j* is usually not empty at the end of stage *i*) and $\lambda_{f,i,j} = 0$ (i.e., reservoir *j* is usually not full at the end of stage *i*),

$$\lambda_{r,i,j} = \lambda_{r,i+1,j} \tag{2.36}$$

If $\lambda_{r,i,j} > 0$, then $\lambda_{r,i+1,j} > 0$, this means that for these two consecutive stages, the reservoir keeps not releasing (Figure 2.5(b)), otherwise, if $\lambda_{r,i,j} = 0$, then $\lambda_{r,i+1,j} = 0$, this means the reservoir should release or possibly not release. (Figure 2.5(a)). This shows that, for any reservoir in the system, the following three conditions might match together as part of the optimization conditions: the MU does not change from stage *i* to stage *i*+1, and the storage is usually neither full nor empty[3] and the operation for the reservoir keeps the same at stage *i* and stage *i*+1, i.e., if the reservoir releases water during stage *i*, it can release water during stage *i*+1, and vice versa.

(2) If $\lambda_{e,i,j}$>0 (i.e., reservoir *j* is empty at stage at the end of stage *i*), and we must have $\lambda_{f,i,j} = 0$ (i.e., the reservoir *j* is not full at the end of stage *i*), then:

$$\lambda_{r,i,j} = \lambda_{r,i+1,j} + \lambda_{e,i,j} \tag{2.37}$$

Therefore,

$$\lambda_{r,i,j} > 0 \tag{2.38}$$

This means that this reservoir can become empty if it does not release at stage *i* (Figure 2.5(c)). This shows that the following three conditions might match together as part of the optimization conditions: the MU does not change from stage *i* to stage *i*+1, the reservoir is empty at the end of stage *i,* and there is no release at stage *i*. This is only possible if there is no inflow during stage *i* and the initial storage of stage *i* is zero. Under such conditions, the reservoir is dry and does not release.

(3) If $\lambda_{e,i,j}$=0 (i.e., reservoir *j* is not empty at the end of stage *i*), and $\lambda_{f,i,j} > 0$ (i.e., reservoir *j* at stage is full at the end of stage *i*),

$$\lambda_{r,i,j} + \lambda_{f,i,j} = \lambda_{r,i+1,j} \tag{2.39}$$

Therefore,

$$\lambda_{r,i+1,j} > 0 \tag{2.40}$$

which means that this reservoir can be full when it has no release at the next stage (Figure 2.5(d)). This means, for any reservoir in the system, the following three conditions might match together as part of the optimization conditions: the MU does not change from stage *i* to stage *i*+1, the reservoir is full at the end of stage *i*, and there is no release at stage *i*+1.

[3] Under the following special cases the reservoir is possibly full or empty: the reservoir is full but there is no benefit associated with keeping more water in the reservoir or empty but there is no benefit associated with releasing more water from the reservoir.

This is only possible if there is no inflow during stage i+1 and the ending storage of stage i+1 is full. Under such conditions, the reservoir is full and does not release.

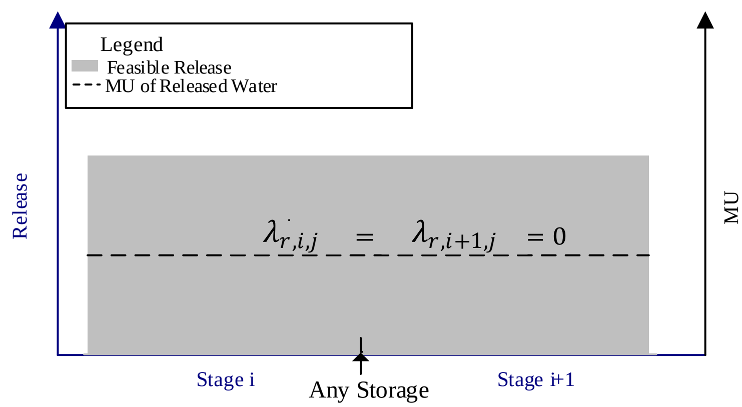
As a summary, with identical MU between stage i and i+1, one of the following situations must occur for any reservoir in the system with the optimal solution:

a. the reservoir is usually neither full nor empty at the end of stage i, and it releases at neither stage;

b. the reservoir is usually neither full nor empty at the end of stage i, and the reservoir can release at both stages.

c. There is no inflow at stage i and the reservoir keeps dry before stage i.

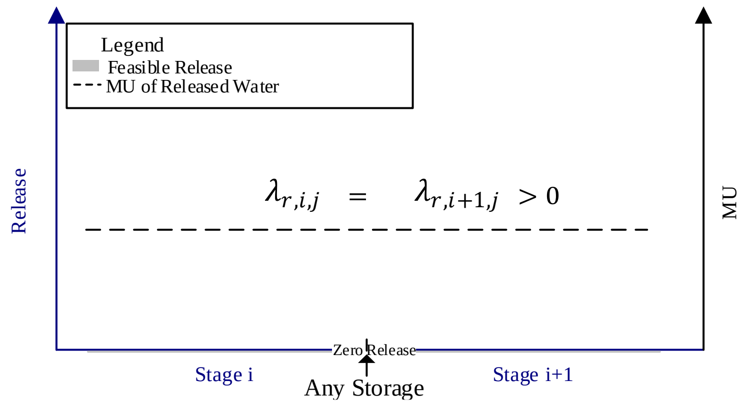
d. there is no inflow at stage i+1 and the reservoir remains full after stage i.



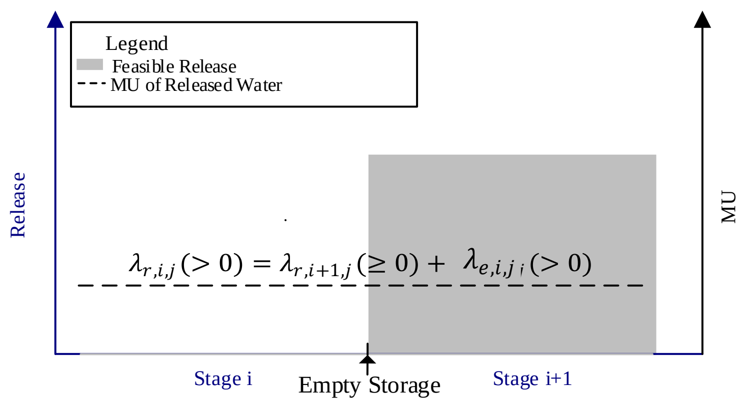
(a) Storage capacity, non-negative storage and non-negative release constraints unbinding

Figure 2.5 Different cases with identical MU

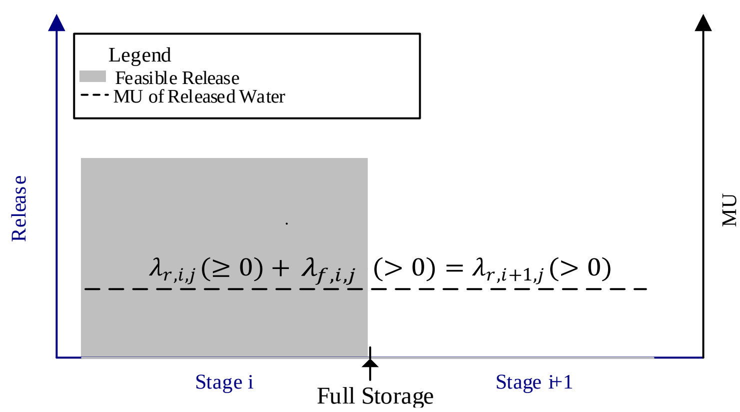
Figure 2.5 (cont.)



(b) Storage capacity, non-negative storage constraints unbinding and non-negative release constraint binding



(c) Non-negative storage constraint binding



(d) Storage capacity constraint binding

Scenarios 1~3 illustrate the local properties of the optimal solution (i.e., from stage i to i+1), as summarized in the following table 2.1

Table 2.1 Local properties of the optimal solution

Scenarios	Cases	Release conditions	Release decisions	Implications
1. Increasing MU	1.$\lambda_{e,i,j}$=0 and $\lambda_{f,i,j} = 0$	$\lambda_{r,i,j} > 0$	No release at stage *i*	Inflow limited; All inflow on stage *i* is reserved for future stages.
	2.$\lambda_{e,i,j}$>0 and $\lambda_{f,i,j} = 0$	$\lambda_{r,i,j} > 0$	No release at stage *i* Empty at the end of stage *i*	No inflow on stage *i*, i.e., inflow limited; Keeps dry.
	3.$\lambda_{e,i,j}$=0 and $\lambda_{f,i,j} > 0$	-	Full at the end of stage *i*	Capacity limited; capacity is fully used to reserve water for the future.
2. Decreasing MU	1.$\lambda_{e,i,j}$=0 and $\lambda_{f,i,j} = 0$	$\lambda_{r,i+1,j} > 0$	No release at stage *i*+1	Inflow limited; No need to empty the storage to reserve capacity for future inflow.
	2.$\lambda_{e,i,j}$>0 and $\lambda_{f,i,j} = 0$	-	Empty at the end of stage *i*	Capacity limited; Storage is emptied to reserve capacity for future inflow.
	3.$\lambda_{e,i,j}$=0 and $\lambda_{f,i,j} > 0$	$\lambda_{r,i+1,j} > 0$	No release at stage *i*+1 Full at the end of stage *i*	No inflow on stage *i*+1, i.e., inflow limited; No need to reserve capacity for future inflow.
3. Identical MU	1.$\lambda_{e,i,j}$=0 and $\lambda_{f,i,j} = 0$	$\lambda_{r,i,j} = \lambda_{r,i+1,j}$	Keep releasing or keep not releasing	The operation for the reservoir keeps the same, i.e., release or not.
	2.$\lambda_{e,i,j}$>0 and $\lambda_{f,i,j} = 0$	$\lambda_{r,i,j} > 0$	No release at stage *i* Empty at the end of stage *i*	No inflow on stage *i*, keeps empty.
	3.$\lambda_{e,i,j}$=0 and $\lambda_{f,i,j} > 0$	$\lambda_{r,i+1,j} > 0$	No release at stage *i*+1 Full at the end of stage *i*	No inflow on stage *i*+1, remains full.

Following the local properties of the optimal solution between stages *i* and stage *i*+1 (as shown in Table 2.1), the properties of the optimal solution for the entire study period (*i*=1, 2, …, N) could be derived. The MU over all the stages can be represented as piece-wise function, with horizontal pieces, representing a number of consecutive stages with identical MU, linked by either increasing pieces (indicating the water demand is becoming less satisfied), or decreasing pieces (indicating the water demand is becoming more satisfied.) (Figure 2.6). In the following, these different types of pieces are discussed, including (I) a piece starting from stage *i* with decreasing MU and ending at stage *i'* where the MU switches to an increasing direction, (II) a piece starting from stage *i* with increasing MU and ending at stage *i'* with increasing MU, (III) piece starting from stage *i* with decreasing MU and ending at stage *i'* with decreasing MU.

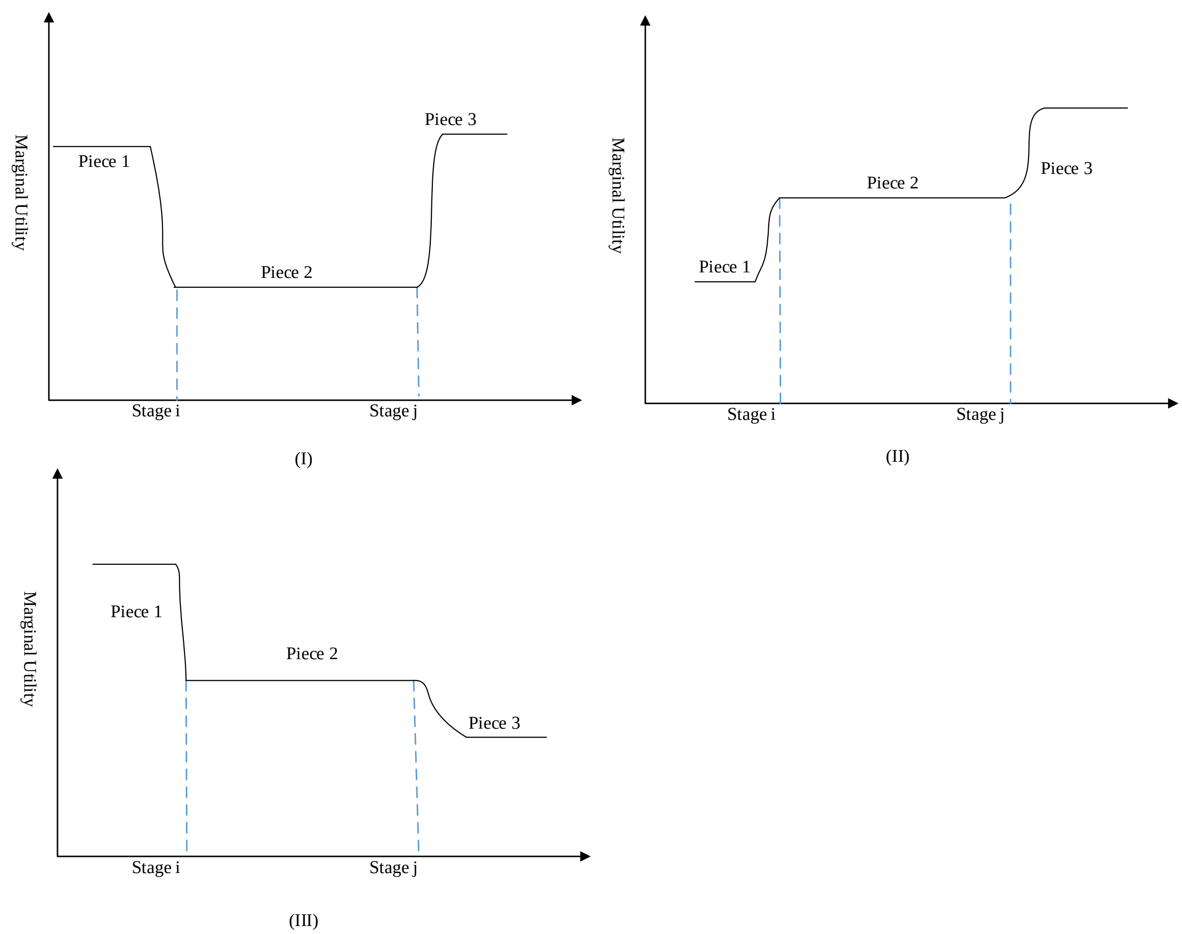


Figure 2.6 Different types of pieces of the optimal MU curve

For (I) (as shown in Figure 2.6(I)), the reservoirs in the system should operate following one of the following policies.

a. Combining Case (2) of Scenario 2 (at stage *i*), Case (3) of Scenario 1 (at stage *i'*) and Case (1) of Scenario 3 for stages between *i & i'*, the reservoir is empty at the end of stage *i* and becomes full at the end of stage *i'*, the reservoir releases over all stages between stage *i* and stage *i'*. This can only occur when the total inflow of the reservoir between stage *i* and stage *i'* is greater than the storage capacity of that reservoir, i.e., the storage capacity limits the reservoir from saving more water to future stages with higher MU.

For case (a) of piece type (I), releases exist along with constant MU piece 2; in the following cases (b, c and d) of piece type (I), we discuss the situations when there is no release along with MU piece 2. For the relationship of the MU on piece 1 and piece 3 (as shown in Figure 2.6), if the change of MU happens within one stage (*i* or *i*'),

$$b'_{i+1}\left(\sum_{j=1}^{M} r_{i+1,j}\right) = b'_{i'}\left(\sum_{j=1}^{M} r_{i,j}\right) \tag{2.41}$$

$$\lambda_{r,i+1,j} = \lambda_{r,i',j} > 0 \tag{2.42}$$

$$\lambda_{r,i',j} + b'_{i'}\left(\sum_{j=1}^{M} r_{i',j}\right) = b'_{i'+1}\left(\sum_{j=1}^{M} r_{i'+1,j}\right) - \lambda_{f,i',j} \tag{2.43}$$

$$b'_{i}\left(\sum_{j=1}^{M} r_{i,j}\right) - \lambda_{e,i,j} = \lambda_{r,i+1,j} + \ b'_{i+1}\left(\sum_{j=1}^{M} r_{i+1,j}\right) \tag{2.44}$$

where, Eq. (2.41) & Eq. (2.42) follow the assumption that the MU is identical on piece 2, and there is no release on piece 2, Eq. (2.43) follows Eq. (2.19) assuming that the reservoir is not empty at the end of the stage i' and that the reservoir releases after the stage i'; Eq. (2.44) follows Eq. (2.27) assuming that the reservoir is not full at the end of stage i and the reservoir releases before the stage i. These two assumptions on storages are reasonable, i.e., the reservoir is not full at the end of stage i and the reservoir is not empty at the end of the stage i', if there exists any inflow on piece 2 (as discussed in Scenario 3).

b. Combining Case (1) & (3) of Scenario 2 (at stage *i*), Case (3) of Scenario 1 (at stage *i'*) and Case (1) of Scenario 3 (all stages between *i & i'*), the reservoir is not empty at stage *i* and is full at stage *i',* and the reservoir has no release during all stages between stage *i* and stage *i'*. Following Eq. (2.41) -(2.44), we have

$$b'_{i'+1}\left(\sum_{j=1}^{M} r_{i'+1,j}\right) = b'_i\left(\sum_{j=1}^{M} r_{i,j}\right) + \lambda_{f,i',j} \tag{2.45}$$

which means the value of water, i.e. marginal benefit, is lower on piece 1 than on piece 3, and thus, for some reservoirs with positive $\lambda_{f,i',j}$, in addition to the water saved from piece 2 to piece 3, the reservoir also saves water from stages of piece 1 to stages of piece 3 as much as possible. This only happens when the total inflow between stage *i* and stage *i'* is less than the storage capacity and the reservoir have excessive storage capacity (i.e. the storage minus the total inflow during stages of piece 2) to save water from earlier stages with higher MU (i.e. stages on piece 1) to the future (i.e. stages on piece 3).

c. Combining Case (2) of Scenario 2 (at stage *i*), Case (1) & (2) of Scenario 1 (at stage *i'*) and Case (1) of Scenario 3 (all stages between *i* & *i'*), the reservoir is empty at stage *i* and is not full at stage *i'*, the reservoir should not release at any stage between stage *i* and stage *i*'. Following Eq. (2.41) -(2.44), we have

$$b'_{i'+1}\left(\sum_{j=1}^{M} r_{i'+1,j}\right) = b'_i\left(\sum_{j=1}^{M} r_{i,j}\right) - \lambda_{e,i,j} \tag{2.46}$$

which means the value of water, i.e. marginal benefit, is higher on piece 1 than that on piece 3, and thus, the reservoir release water as much as possible during stages of piece 1. This happens when the total inflow between stage *i* and stage *i'* is less than the storage capacity. However, under this case, though there is excessive storage capacity at the end of piece 2, the reservoir should only save water from piece 2 to piece 3 but not piece 1, since the MU is lower on piece 3 than that on piece 1.

d. Combining Case (1) & (3) of Scenario 2 (at stage *i*), Case (1) & (2) (at stage *i'*) of Scenario 1, and Case (1) of Scenario 3 (all stages between *i* & *i'*), the reservoir is not empty at the end of stage *i* and is not full at the end of stage *i'*, the reservoir should not release at any stage between stage *i* and stage *i'*. Following Eq. (2.41) -(2.44), we have

$$b'_{i'+1}\left(\sum_{j=1}^{M} r_{i'+1,j}\right) = b'_i\left(\sum_{j=1}^{M} r_{i,j}\right) \tag{2.47}$$

which means an identical MU is achieved between piece 1 and 3. This only happens when the total inflow between stage *i* and stage *i'* is less than the storage capacity. Under this case, the excessive storage capability is large enough to regulate water inflow between piece 1 and piece 3. As a result, some water is saved from piece 1 to piece 3 to achieve identical MU.

The situation of MU change over some stages, which is not covered above, can be as follows: piece type (III), (I) and (II) happen consecutively. Under this situation, there might be a number of decreases of the MU before it reaches a minimum value and then increases over a number of stages. Similar to the results from Eq. (2.41) – (2.44), it can be proved that, if a reservoir in the system between stage i and i'satisfies: (1) stage i is part of piece type (III) with decreasing MU and stage i' is part of piece type (II) with increasing MU; (2) the reservoir does not release between stage i and stage i' but releases before stage i and after stage i', then the MU is the higher on the piece before stage i if the storage of the reservoir is empty at the end of stage i; the MU is higher on the piece after stage i' if the storage of the reservoir is full at the end of stage i'; or the MUs are identical at the stages before stage i and after stage i' if the reservoir is neither full at the end of stage i' nor empty at the end of stage i.

For (II) (as shown in Figure 2.6(II)), reservoirs in the system should operate following one of the following policy.

a. Combing Case (3) of Scenario 1 (at stage *i* & *i'*), and Case (1) of Scenario 3 (all stages between *i* & *i'*), the reservoir is full at the end of both stage *i* and stage *i'*. This only happens when the storage at the beginning of piece 1 plus the total inflow during the stages of piece 1 is larger than the storage capacity of the reservoir. The capacity limits the reservoir from saving more water to future stages with higher MU.

b. Combing Case (1) & (2) of Scenario 1 (at stage *i*), Case (3) of Scenario 1 (at stage *i'*) and Case (1) of Scenario 3 (all stages between *i* & *i'*), the reservoir is not full at the end of stage *i* and is full at the end of stage *i'* and the reservoir does not release at any stage of piece 1. This only happens when the storage at the beginning of piece 1 plus the total inflow during the stages of piece 1 is less than the storage capacity of the reservoir, and the storage at the beginning of piece 2 plus the total inflow during stages of piece 2 is larger than the storage capacity of the reservoir. Thus, to save water for future stages with higher MU, i.e. stages of piece 3, the reservoir is full at the end of stage *i'*, and has a preference to save water first from stages of piece 1 with a lower MU than that of piece 2.

c. Combing Case (1) & (2) of Scenario 1 (at stage *i*& *i'*) and Case (1) of Scenario 3 (all stages between *i* & *i'*), the reservoir is not full at the end of both stage *i* and stage *i'* and the reservoir does not release at any stage of piece 1 and piece 2. This only happens when the storage at the beginning piece 1 plus the total inflow during the stages of piece 1 and piece 2 is smaller than the storage capacity of the reservoir, thus all inflow during the stages of piece 1 and piece 2 are saved for the future with higher MU.

For (III) (as shown in Figure 2.6(II)), reservoirs in the system should operate following one of the following policy.

a. Combining Case (2) of Scenario 2 (at stage *i*& *i'*) and Case (1) of Scenario 3 (all stages between *i* & *i'*), the reservoir is empty at the end of both stage *i* and stage *i'*. This indicates that there is abundant inflow during stages after stage *i'*, so that all water should be released to make space for the coming inflow.

b. Combining Case (2) of Scenario 2 (at stage *i*), Case (1) & (3) of Scenario 2 (at stage *i'*) and Case (1) of Scenario 3 (all stages between *i* & *i'*), the reservoir is empty at the end of stage *i,* and is not empty at the end of stage *i',* and does not release at any stage of piece 3. This indicates that the capacity of the reservoir is large enough to save some inflow during the stages of piece 2 and all inflow during the stages of piece 3 to a future stage with MU greater or equal to that of piece 2 but smaller than that of piece 1.

c. Combining Case (1) & (3) of Scenario 2 (at stage *i* & *i'*) and Case (1) of Scenario 3 (all stages between *i* & *i'*), the reservoir is not empty at the end of both stage *i* and stage *i',* and does not release at any stage of both piece 2 and piece 3. This indicates the capacity of the reservoir is large enough to save some inflow during the stages of piece 1 and all inflow during the stages of piece 2 & 3 to a future stage with MU greater than or equal to that of piece 1.

The basic idea of reservoir operation is to save water during water abundant stages when the MU is lower to water stress stages when the MU is higher. Ideally, if the reservoir storage is large enough, the MU should be identical over stages with an optimal solution. However, as the reservoir capacity is limited, the actual optimal solution will result in full-empty cycles of storage with identical MU between two consecutive full-empty stages and MU changes at empty or full states, i.e., the reservoir releases to empty when the MU decreases and the reservoir stores water to full when the MU increases. Similarly, for a system with multiple reservoirs in parallel, full-empty cycles exist as a result of the limited storage capacity of the reservoirs in the system, i.e., a full storage state of a reservoir might match together with increasing MU as part of the optimal operation policy; or an empty state of a reservoir might match together with a decreasing MU. However, according to our analysis, other conditions, i.e., no release before the MU increases or after the MU decreases can also match together with the changes of the MU as part of the optimal policy. It can be showed that at least one reservoir in the system will have either full or empty storage for the increased or decreased MU, i.e., the storage is the limiting factor which prevents the reservoir from saving more water from one to another stage to achieve an identical MU as required by the economic principle. That is to say, the fundamental cause of the change of MU is the capacity of the reservoirs. However, due to the diversity of reservoirs in the system, there might also exist some reservoirs with extra-large capacity or relatively small inflow, such that the capacity of the reservoir is not limiting the reservoir from saving water to a future stage. As a result, all inflow of those reservoirs can be saved during stages with high inflow or low demand. Such reservoirs, as shown in Eq. (2.41) – (2.44), can regulate water over a longer period than the reservoirs with a limited capacity; they have less frequent full-empty cycles. Therefore, in a system with multiple reservoirs in parallel, the fundamental cause for MU changes is the limited capacity, while the difference in capacity and inflow of reservoirs in the system leads to different water regulation abilities, which results in asynchronized full-empty cycles among reservoirs. On the other hand, though asynchronized full-empty cycles might exist among different reservoirs, for those reservoirs that do not become full or empty at certain stages, it must be have a capacity that is large enough to store all water during stages with high inflow or low demand, to achieve optimality.

Ideally, if all reservoirs in the system have the same ability to regulate inflow, they will produce synchronized full-empty cycles as if they are combined into a single reservoir. Based on principles of optimization, the optimal operation policy for such a virtual single reservoir (which is equivalent to an optimization problem obtained from original optimization problem by removing the constraint set by the multiple reservoir fact) should be the best possible optimal solution that the actual parallel reservoir system could achieve when considering the fact of multi-reservoir system. However, for some reservoirs with extreme large capacity or extreme small inflow, it is impossible for them to become full even if they do not release any water before the full storage states of other reservoirs and, for the same reason, these reservoirs might also not need to synchronize their empty storage stage with other reservoirs. Under these situations, the actual optimal solution will deviate from that resulting from the virtual single reservoir.

As a summary, the optimal operation policy for a reservoir system in parallel have the following properties: (1) the MU evaluated at the demand site is a piece-wise function, with horizontal pieces, representing a number of consecutive stages with identical MUs, linked by either increasing pieces (indicating the water demand is becoming less satisfied) and decreasing pieces (indicating the water demand is becoming more satisfied). The water demand satisfactory level is affected by various factors including demand levels, inflow conditions, and reservoir storage capacities. (2) Based on the relationship between the inflow and the storage capacity, different reservoirs behave differently when the MU changes; (3) for reservoirs with relatively small inflow and large capacity, if they do not release at all during a number of consecutive wet stages with high inflow or low demand, they can be not full when the MU increases after the wet stages or not empty when the MU decreases before the wet stages, as they have sufficient capacity to save all inflow during the wet stages to future dry stages with low inflow or high demand; specially, these reservoirs can regulate water between dry stages with higher MU before and after a number of consecutive wet stages with lower MU, the maximum amount of water can be saved from the earlier dry stage to the later dry stage is the excessive storage capacity of the reservoirs after storing as much water as possible during the wet stages; (4) for normal reservoirs (reservoirs other than those in (3)), they become full when the MU increases and become empty when the MU declines, which is due to the limitation of their capacity.

2.3 Algorithm Development

Any solution that satisfies the KKT conditions (Eq. (2.6) – Eq. (2.14)) will be the optimal solution for the reservoir systems with a concave objective function and linear constraints. Following this property, we design an algorithm to solve the parallel reservoir system problem.

Based on the discussion above, we solve the optimization problem under the assumption (Assumption I) that all reservoirs will release as much water as possible before a decrease in MU, i.e., all reservoir storages achieve the minimum possible value when the MU decreases, and do not save water for future droughts. The following properties (see proofs in Appendix B) from the theoretical analysis are used to develop the solution algorithm: (1) if all reservoirs become full at the first occurrence of MU increase in the study horizon with the optimal solution solved with Assumption I, then, the optimal release decision before the reservoirs are full solved with Assumption I will not be subject to Assumption I. This is because, under this case, there is no reservoir with extra capacity to save water for the future at the first occurrence of MU increase; thus, no more water can be saved from stages before the first occurrence of the MU increase to the future; (2) If in the optimal decisions solved with Assumption I, there is no decrease of MU before the first occurrence of MU increase (i.e., there is a single MU before the first occurrence of the MU increase), then, the optimal release decision before the first occurrence of MU increase will be the same as the optimal release decision of the original problem that is not subject to Assumption I. This is because under this condition, with the optimal decision solved under Assumption I, at the first occurrence of the MU increase, reservoirs are either full or not releasing at all before that stage, so as much as water is saved before the first occurrence of the increase of the MU, and no more water can be saved for future droughts even with reservoirs having extra capacity; (3) if no states with increasing MU happens in the optimal release decision solved with Assumption I, then the optimal release decision will be the same as the optimal decision of the original problem that is not subject to Assumption I. This is because under this case, the MU is non-increasing through the whole study horizon; thus, after removing Assumption I from the optimal release decision solved under Assumption I, there is no need to save water for the future when MU decreases, as much water as possible should be used in the current periods which are drier compared to the future. (4) if neither the condition of (1), (2) nor (3) are satisfied in the optimal release decision under Assumption I, then the optimal release decision between the first occurrence of the MU increase and the adjacent previous occurrence of the decrease of the MU will be the same as the optimal release decision of the original problem that is not subject to Assumption I. Under this case, after removing Assumption I from the optimal release decision solved under Assumption I, though water might be saved from the stages before the adjacent occurrence of the MU decrease to the stages after the first occurrence of the MU increase, the water will not be used between the first occurrence of the MU increase and the adjacent previous occurrence of the MU decrease as this period has the lowest MU of water before the first occurrence of increasing MU.

Based on the analysis above, an algorithm is developed to solve the optimal solution of a system of reservoirs in parallel, as shown below:

(1) Assume all reservoirs will release as much water as possible before a decrease in MU, i.e., all reservoir storage achieves a minimum possible value when the MU decreases (Assumption I), that is, all reservoirs release as much as possible for the current stage without considering future demand. Following the analysis above, a reservoir in the system either becomes full when the MU increases, or does not release during the water abundant stages before the MU increases. Solve for the optimal solution up to the first occurrence of the increase in MU.

Sub-steps under Step (1) under Assumption I are described as follows.

(1-1) Starting from the first stage, calculate the range of the MU $[\underline{\lambda}_i, \bar{\lambda}_i]$, where $\underline{\lambda}_i$ refers to the ending storages specified by Eq. (2.48), i.e., the storage at stage i is the storage capacity of the reservoir if the total water available up to stage i is greater or equal to the capacity, otherwise, the storage is set to the total inflow from the first stage to stage i, and $\bar{\lambda}_i$ refers to the ending storages specified by Eq. (2.49), i.e., the lowest possible storage while ensuring the ending storage constraints are not violated. Especially, for the last stage N, there is a single value for λ with a given ending storage.

$$s_{i,j} = \begin{cases} K_j & if \sum_{k=1}^{i} I_{k,j} + s_{0,j} \geq K_j \\ \sum_{k=1}^{i} I_{k,j} + s_{0,j} & if \sum_{k=1}^{i} I_{k,j} + s_{0,j} < K_j \end{cases},$$

$$for\ i = 1,2, \dots, N-1, j = 1,2, \dots M \quad (2.48)$$

$$s_{i,j} = \max\left\{0, s_{N,j} - \sum_{k=i+1}^{N} I_{k,j}\right\}, \qquad for\ i = 1,2, \dots, N-1, j = 1,2, \dots M \quad (2.49)$$

For both cases, the utility is identical before stage i, i.e., $\underline{\lambda}_i$ and $\bar{\lambda}_i$ are the solution of Eq. (2.50) & Eq. (2.51),

$$b_1'(r_1) = b_2'(r_2) = \cdots = b_i'(r_i^*) = \lambda_i \quad (2.50)$$

$$\sum_{k=1}^{i} r_k = \sum_{k=1}^{i} \sum_{j=1}^{M} I_{k,j} + \sum_{j=1}^{M} s_{0,j} - \sum_{j=1}^{M} s_{i,j} \quad (2.51)$$

where λ_i is either $\underline{\lambda}_i$ or $\bar{\lambda}_i$ depending on the storage at the end of stage *i* as specified above, r_i is the total release of all reservoirs at stage *i* (Eq. (2.52)).

$$r_i = \sum_{j=1}^{M} r_{i,j}, \qquad for\ i = 1,2, \dots, N \quad (2.52)$$

(1-2) Starting from the first stage, identify the intersection of the ranges of the MU before stage *i*, $\bigcap_{k=1}^{i} [\underline{\lambda}_k, \bar{\lambda}_k]$. As $\bigcap_{k=1}^{i} [\underline{\lambda}_k, \bar{\lambda}_k] \subset [\underline{\lambda}_k, \bar{\lambda}_k]$ for $k = 1,2, \dots, i$, thus, a uniform MU in the range of $\bigcap_{k=1}^{i} [\underline{\lambda}_k, \bar{\lambda}_k]$ for all stages from the first stage to stage *i* can ensure that the storage of any reservoir storage falls in a feasible range (i.e., between 0 and the storage capacity, and ensuring the ending storage constraint not violated) for $k = 1,2, \dots, i$.
At stage *i*, if

$$\bigcap_{\mathrm{k}=1}^{\mathrm{i}} [\underline{\lambda}_k, \bar{\lambda}_k] = \Phi \quad (2.53)$$

then, there does not exist any single MU value over all stages before i and thus either the empty or full storage constraint is binding between the beginning and stage i. Then, one of the following cases (a-c) will happen.

(a) If Eq. (2.53) is first satisfied at stage *i* and $\underline{\lambda}_i > \min\{\bar{\lambda}_1, \bar{\lambda}_2, \dots, \bar{\lambda}_{i-1}\}$, there does not exist any single MU value over all stages before i, and MU increases at some stages before i. Assume the first occurrence of MU increase happens at the end of stage i_f, then, before i_f there are stages with high inflow compared to demand and after i_f there are stages with low inflow compared to demand. Thus, water should be saved as much as possible during the stages with high inflow, and correspondingly, the MU before i_f should be as high as possible. Thus, we have $\bar{\lambda}_{i_f} = \min\{\bar{\lambda}_1, \bar{\lambda}_2, \dots, \bar{\lambda}_{i-1}\}$, and thus, i_f can be determined. Then, the optimal system level release decision before i_f can be calculated using Eq. (2.48) – (2.52) by applying these equations from the beginning to i_f, and the storages at the end of i_f are specified by Eq. (2.48).

(b) If Eq. (2.53) is satisfied and $\bar{\lambda}_i < \max\{\underline{\lambda}_1, \underline{\lambda}_2, \dots, \underline{\lambda}_{i-1}\}$, there does not exist any single MU value over all stages before i, and MU decreases at some stages before i. Assume the first occurrence of the decrease of the MU happens at the end of stage i_e, then, before i_e there are stages with low inflow compared to demand and after i_e there are stages with high inflow compared to demand. Thus, water should be released as much as possible during the stages with low inflow, and correspondingly, the MU before i_e should be as low as possible. Thus, we have $\bar{\lambda}_{i_e} = \min\{\bar{\lambda}_1, \bar{\lambda}_2, \dots, \bar{\lambda}_{i-1}\}$, and thus, i_e can be determined. Then, the optimal system level release decision before i_e can be calculated using Eq. (2.48) – (2.52) by applying theses equations from the beginning to i_e, and the storages at the end of i_e are specified by Eq. (2.49).

(c) Otherwise, if Eq. (2.53) is not satisfied at all stages, a single MU could be obtained at all stages without violating any constraints. Then, the optimal system level release decision can be calculated using Eq. (2.50) – (2.52) by applying theses equations from the beginning to last stage.

(1-3) Under case (a), release decision up to the first occurrence of the increase of the MU can be determined, and continues to step (2); under case (b), release decision up to the first occurrence of the decrease of the MU can be determined (at the end of stage i_e), and the release decision for the rest of the study period will be solved by solving the problem from stage $i_e + 1$ to the end of the study horizon again using the algorithm in step (1) and setting the initial storages as empty, until the first occurrence of the increase of the MU is determined or the last stage is achieved; under case (c), entire release decisions are solved.

(2) Based on the solution solved from step (1) under Assumption I, one of the following steps will be undertaken. These steps follow the discussion at the beginning of this section.

(a) If there is a stage (i) with constant MU over all stages before the stage (i), break the entire study horizon into two parts, part 1 covers stages 1 to i; part 2 covers the rest. The optimal solution of part 1 will be the same as the optimal solution solved from the problem under Assumption I. For part 2, set the initial storage of all reservoirs as the corresponding storage at the time when the MU is first increased in the optimal solution solved from the problem under Assumption I, and set the new study horizon from the first occurrence of the increase of the MU to the end of the study horizon, and solve the problem again using this algorithm.

(b) if there is a stage with increasing MU and all reservoirs are full at the first occurrence of MU increase, then, break the entire study horizon into two parts, stages before the first occurrence of MU increase (part 1) and the rest of study period (part 2). The optimal solution of part 1 will be the same as the optimal solution solved from the problem under Assumption I. For part 2, set the initial storage of all reservoirs as their capacity, and set the new study horizon from the first occurrence of the MU increase to the end of the original study horizon, and solve the problem again using this algorithm.

(c) if the final stage is achieved without increasing MU at any stage, then, the release decision will be the same as that under Assumption I, and the algorithm stops.

(d) if none of (a) – (c) satisfies, then the optimal release decision between the first occurrence of the increasing MU and the adjacent previous occurrence of the decreasing MU can be determined, and the release decisions at other stages are solved by solving the following problem again using the algorithm. Assuming the first occurrence of an increasing MU happens at the end of stage i_f and the adjacent previous occurrence of a decrease MU happens at the end of stage i_e, the problems is stated as follows: Take the stages from the beginning to the end of i_e and the stages from the end of i_f to the end of the study period (which is defined as a sub-study horizon). Set the storage capacity at the end of i_e as the remaining storage capacity defined as Eq. (2.54) and set the inflow at the stage after i_e in the new problem, which is the stage after i_f in the original problem as Eq.

(2.55),

$$K_{i_e,j}^{new} = K_j^{ori} - s_{i_f,j}^{A} \tag{2.54}$$

$$I_{i_e+1,j}^{new} = I_{i_f+1,j}^{ori} + s_{i_f,j}^{A} \tag{2.55}$$

where, $K_{i_e,j}^{new}$, $I_{i_e+1,j}^{new}$ are the storage capacity of reservoir j at the end of i_e and the inflow at the stage after i_e to reservoir j in the new problem respectively, K_j^{ori} and $I_{i_f+1,j}^{ori}$ are the storage capacity of reservoir j and the inflow at the stage after i_f to reservoir j in the original problem, respectively, and $s_{i_f,j}^{A}$ is the ending storage of reservoir j at the end of stage i_f in the optimal solution of the original problem under Assumption I. This step follows the fact that the optimal release decision between the first occurrence of an increasing MU and the adjacent previous occurrence of a decreasing MU in the optimal solution under Assumption I will be the same as the optimal release decision of the original problem without Assumption I, and also that if there are two periods (P1 and P2) with higher MU before and after a period (P3) with lower MU, and there is remaining storage capacity at the end of P3, i.e., some reservoirs are not full at the end of P3, then P1 and P2 will achieve a uniform MU by saving water from P1 to P2, but the maximum amount of water saved from P1 and P2 is constrained by the remaining storage capacity at the end of P3.

(3) After step 2, the system level releases are determined, and also, we determine full and empty states for all reservoirs, and thus we can determine the total release of each reservoir between each pair of consecutive full/empty states by simply applying the mass balance relationship. As long as the operation activity follows these two requirements, the release decisions are always optimal. As we explicitly consider the constraints when solving for the optimal solution, it is ensured that a feasible solution will exist.

2.4 Case Study

To demonstrate the solution algorithm applied to the operation of a parallel reservoir system, a simple synthetic case is presented as follows. Inflow data for a system consisting of 5 parallel reservoirs is synthesized by the Thomas-Fiering model (Thomas & Fiering, 1962),

$$q_{i+1} = \mu + \rho_{flow}(q_i - \mu) + \sqrt{1 - \rho_{flow}^2}(\mu C_v)\omega \tag{2.56}$$

where q_i is the inflow at stage i, μ is the average inflow an is set as 1 for reservoirs 1-4 and 0.1 for reservoir 5, which is also set as the initial inflow at the first stage, ρ_{flow} represents the temporal correlation of the inflow and is set as 0.3 for all reservoirs, and C_v represents the inflow variability and is set as 0.5 for all reservoirs, and ω is a random number generated from standard normal distribution. For simplicity, a constant demand 1 is used at all stages, a uniform storage capacity 2 is applied to all reservoirs, and the MU function $b'(\cdot)$ is assumed to be a function of release/demand ratio with the same form at all stages, i.e.,

$$b_i{}'(r_i) = b'\left(\frac{r_i}{d_i}\right) \tag{2.57}$$

where, r_i is the release at stage i, d_i is the demand at stage i.

Figure 2.7 shows the optimal release schedule given by the algorithm. The top figure shows the MU over stages, and the middle figure shows the system level releases over stages, and the bottom figure shows the individual reservoir release schedule, which consists of empty states (blue stars), full states (red circles), no release stages (grey color), and the total release (numbers) between two consecutive empty/full states. As can be seen, all reservoirs become empty at stage 21, when the MU decreases, and all reservoirs become full at stage 33, when the MU increases, and reservoirs 1-4 become full at stage 31 and reservoir 5 does not release before stage 31, which is a stage with an increasing MU. Furthermore, reservoir 5 is empty at stage 21 and not full at stage 31 with no release between the two stages, which corresponds to the higher MU at stage 20 than that at stage 32. The results from the algorithm satisfy all the requirements derived from the theoretical analysis.

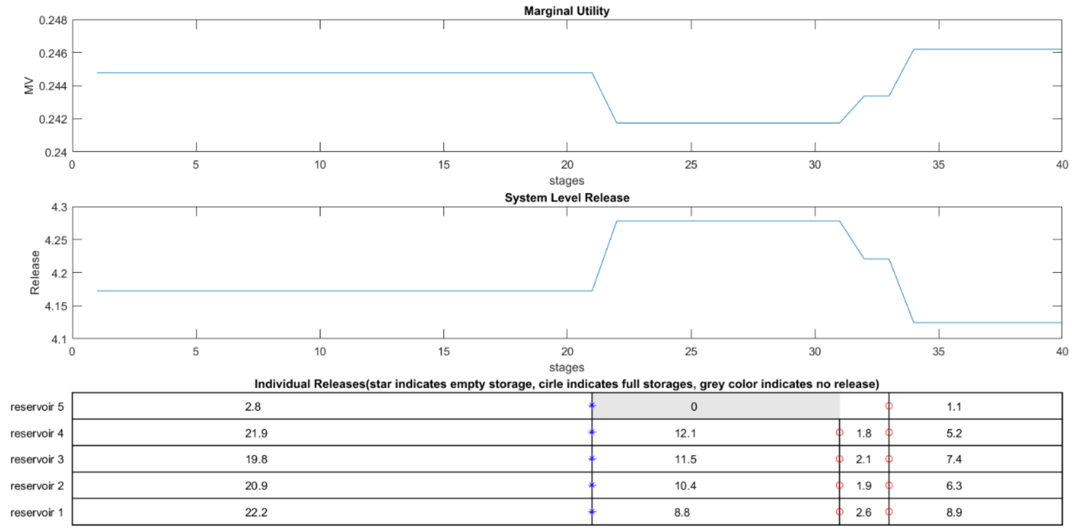
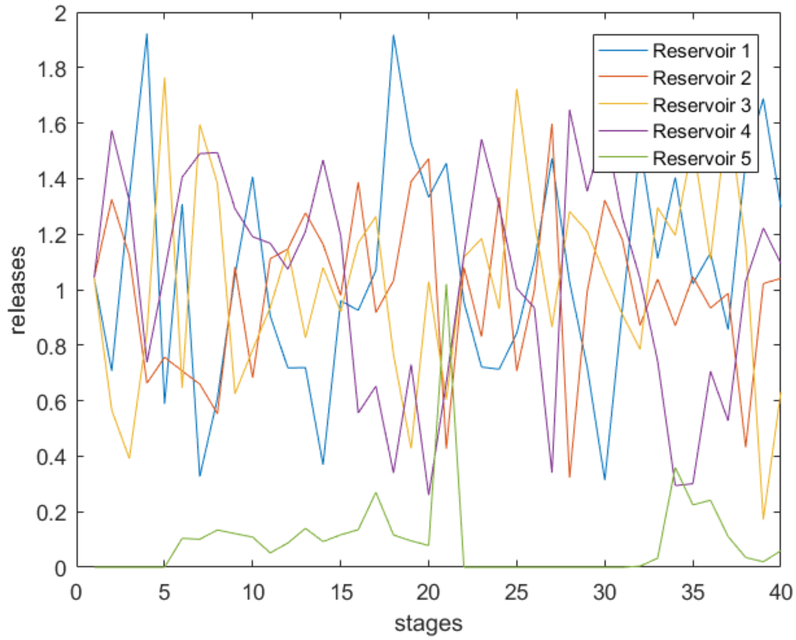
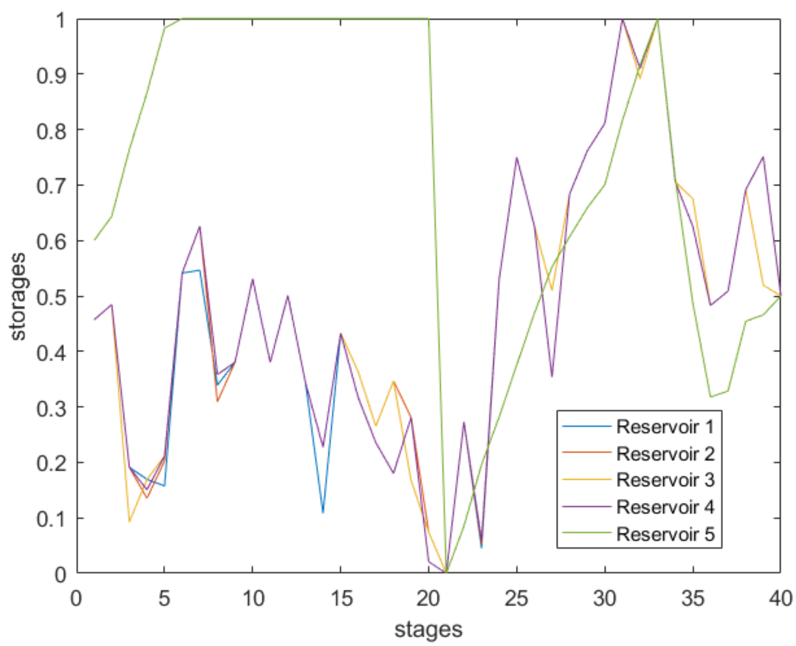


Figure 2.7 Optimal release schedule given by the algorithm (top figure: MU over stages, middle figure: system level releases over stages, bottom figure: individual reservoir release schedule)

Based on the optimal release schedule shown in Figure 2.7, the system level release schedule can be further broken down into individual release schedules. As there usually exist numerous optimal individual release schedules, reservoir operators might decide their own release schedule based on their own preference. For this case study, we derive the individual release schedule according to the following rules: (1) reservoirs with shorter full-empty cycle has a higher priority to release, i.e., in this case study, reservoir 1-4 release first before reservoir 5; (2) for the reservoirs with the same release priority, the release from each reservoir is proportional to the total available water of that reservoir. Following these two rules, the individual reservoir releases are determined as Figure 2.8(a), and the storages shown in 2.8(b). Further, as shown in Figure 2.9, the system level release given by the individual releases shown in Figure 2.8(a) provides the same system level release as required by the optimal solution given by the algorithm. Thus, individual releases shown in Figure 2.8(a) actually provides one set of optimal individual release decisions. Note these two rules do not always guarantee one set of feasible optimal solution, especially when there exist extremely high inflows to some reservoirs, but with a few more adjustments, a set of feasible solution can be obtained.



(a) Individual releases



(b) Individual storages

Figure 2.8 Individual releases and storages

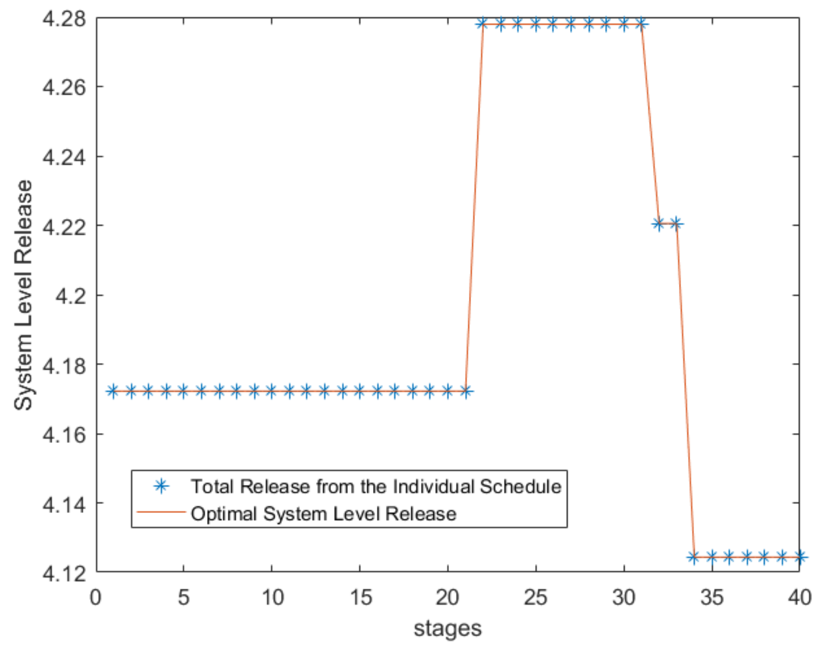


Figure 2.9 Validation of the individual releases

2.5 Conclusions

In this chapter, a multi-stage optimization model is set up to derive the properties of the optimal release decision for a system of reservoirs in parallel with a single demand site. Analytical analysis is conducted using KKT conditions. We show that the optimal operation policy for such as a reservoir system has the following properties: (1) the MU evaluated at the demand site changes with time according to the inflow condition, demand level, etc., in a piece-wise form. (2) based on the relationship between the inflow and the storage capacity, different reservoirs behave differently when the MU changes; (3) for reservoirs with relatively small inflow and large capacity, if they do not release at all during wet stages with low demand, they can be not full when the MU increases after the wet stages or not empty when the MU decreases before the wet stages, as they have sufficient capacity to save water from the wet stages to future dry stages with high demand; specially, these reservoirs can regulate water between dry stages with higher MU before and after a number of consecutive wet stages with lower MU, the maximum amount of water can be saved from the earlier dry stage to the later dry stage is the remaining storage capacity of the reservoirs after storing as much water as possible during the wet stages; (4) for normal reservoirs (reservoirs other than those in (3)), they become full when the MU increases empty when the MU decreases, which is due to the limitation of their capacity.

Furthermore, based on the understanding of the properties of the optimal solution for a system of reservoirs in parallel, an algorithm is developed to solve the optimization model defined for such a system with a single demand site. The algorithm follows the properties derived from the KKT conditions. The algorithm can be applied to solve parallel reservoir operation problems with computational efficiency. A synthetic case study is conducted to show the application of the algorithm with effectiveness and validity.

CHAPTER 3: DETERMINING EFFECTIVE FORECAST HORIZON

3.1 Introduction

Inflow forecast is important for making reservoir operation decisions, while imperfect forecast might significantly reduce the usefulness of the forecast information (Mishalani and Palmer, 1988). Among the previous studies, You and Cai (2008) developed a theoretical relationship based on dimensional analysis for determining the forecast horizon, which is a length of the forecast beyond which the inflow will no longer affect the release decision in the decision horizon. Zhao et al. (2012) conducted numerical experiments with imperfect forecast and proposed the concept of effective forecast horizon with a certain level of uncertainty, which provides maximum information to support reservoir operation decisions. In general, the effective forecast horizon becomes shorter as forecast uncertainty increases. This study continues the efforts listed above. We analyze the optimal release decision using a multi-stage reservoir operation model and develop numerical procedures to determine the effective forecast horizon (EFH). This study enables reservoir operation decision makers to fully use inflow forecast while controlling the forecast uncertainty.

In this work, we follow the following procedures to develop the criteria and procedures to identify the EFH under a given uncertainty level. First, we set up a deterministic model to discuss the characteristics of optimal multistage reservoir operation solutions. Then, the relationship between the uncertain inflow forecast and the optimal release decisions are explored, i.e., the lower bound and upper bound of the optimal release decisions over all possible inflow scenarios. Following that, we will establish a criterion to examine whether the forecast heading period of any given forecast can be a candidate of the EFH or not. Finally, the overall procedures for determining EFH are proposed.

3.2 The Mathematical Model

A deterministic optimization model is first set up for multi-stage reservoir operation to optimize the utility function over all time period with mass balance, non-negative storage and reservoir capacity as constraints (Draper and Lund, 2004; You and Cai, 2008a, 2008b). The results of the deterministic optimization model will be used to prove the criteria to be established for the EFH and to design an algorithm to determine the EFH considering inflow uncertainty. The solution of this model provides the “ideal” optimal release decision for the formulated problem. For simplicity, water loss from the reservoir such as evaporation and leakage are ignored in the analysis without loss of significance.

$$\text{Obj. } \max \sum b_i(r_i) \tag{3.1}$$

$$s.t.$$

$$s_{i-1} + I_i - r_i = s_i, \qquad for\ i = 1,2,\dots,N \tag{3.2}$$

$$s_i \geq 0, \qquad for\ i = 1,2,\dots,N \tag{3.3}$$

$$s_i \leq K, \qquad for\ i = 1,2,\dots,N \tag{3.4}$$

where $b_i(r_i)$ is the concave utility function for stage i, s_i is the storge of reservoir at the end of stage i, I_i is the inflow forecast during stage i, r_i is the release during stage i, K is the capacity of the reservoir and N is the attempted forecast horizon.

Applying the KKT conditions (Bazaraa et al., 2013) to the model described above, we obtain the following general conditions for an optimal solution (see Appendix C for detailed derivation.)

$$-\frac{\partial b_i(r_i)}{\partial r_i} + \lambda_{b,i} = 0, \qquad for\ i = 1,2,\dots,N \tag{3.5}$$

$$\lambda_{b,i-1} - \lambda_{b,i} - \lambda_{e,i} + \lambda_{f,i} = 0, \qquad for\ i = 2,\dots,N \tag{3.6}$$

where $\lambda_{e,i}$ is the marginal price for non-negative storage constraints, $\lambda_{f,i}$ is the marginal price for capacity constraints and $\lambda_{b,i}$ is the marginal price for mass balance constraints.

In the following, we discuss the various forms of Eq. (3.5) and (3.6) under different conditions of binding/unbinding constraints. We use the cumulative inflow curve and cumulative water delivery curve, which are used in the classic Rippl’s curve (Ripple, 1883), to illustrate the conditions and associated optimal releases. Three cases for stage *i* and stage *i*+1 are discussed, including both capacity and non-negative storage constraints unbinding, the non-negative storage constraint binding, and the storage capacity constraint binding.

Case 1: Both capacity and non-negative storage constraints are unbinding

With unbinding capacity and non-negative storage constraints,

$$\lambda_{e,i} = 0 \tag{3.7}$$

$$\lambda_{f,i} = 0 \tag{3.8}$$

Substitute Eq. (3.7) and (3.8) to Eq. (3.5) and (3.6),

$$b_i'(r_i^*) = \frac{\partial b_i}{\partial r_i} = \lambda_{b,i} \tag{3.9}$$

$$b_i'(r_i^*) = \lambda_{b,i} = \lambda_{b,i+1} = b_{i+1}'(r_{i+1}^*) \tag{3.10}$$

which means that the marginal utility (MU), i.e. economic value of water, which might be affected by several factors including the inflow conditions and the demand, are identical between stage *i* and stage *i*+1. Under this scenario, the reservoir is large enough to regulate the inflow to the reservoir to match the demands on both stages, and the economic principal is followed, i.e. identical MU between two stages (Figure 3.1(a)).

Case 2: Only the non-negative storage constraint is binding

Assuming the non-negative storage constraints become binding at stage *i*,

$$\lambda_{e,i} > 0 \tag{3.11}$$

and the capacity constraint is unbinding,

$$\lambda_{f,i} = 0 \tag{3.12}$$

Substitute Eq. (3.11) and (3.12) to Eq. (3.5) and (3.6),

$$\lambda_{b,i} = \lambda_{b,i+1} + \lambda_{e,i} \tag{3.13}$$

$$b_i'(r_i^*) = b_{i+1}'(r_{i+1}^*) + \lambda_{e,i} \tag{3.14}$$

According to Eq. (3.14), with the optimal solution, the marginal utility (MU), i.e. the economic value of water, will decrease right after the empty storage state, indicating the system state shifting from a water short stage, i.e., water supply is limited compared to water demand, to a water abundant stage, i.e., water supply is sufficient compared to water demand. The relatively low inflow or relatively high demand at stage *i* require the reservoir to release as much water as possible, and thus the reservoir storage reaches to the empty state at the end of stage *i* (Figure 3.1(b)), associated with a relatively large MU before the empty state.

Case 3: Only the storage capacity constraint is binding

Assuming the storage capacity constraints become binding at stage i,

$$\lambda_{f,i} > 0 \tag{3.15}$$

and the non-negative constraint is unbinding, i.e.,

$$\lambda_{e,i} = 0 \tag{3.16}$$

Substitute Eq. (3.15) and (3.16) to Eq. (3.5) and (3.6),

$$\lambda_{b,i-1} = \lambda_{b,i} - \lambda_{f,i} \tag{3.17}$$

$$b'_{i-1}(r^*_{i-1}) = b'_i(r^*_i) - \lambda_{f,i} \tag{3.18}$$

Under the optimal release decision, the MU will increase right after full storage occurs, indicating the system shifts from a water abundant stage, where the water supply is sufficient compared to water demand, to a water short stage, where the water supply is limited compared to water demand. The relatively high inflow or the relatively low demand at stage *i* require the reservoir to store as much water as possible to stage *i*+1, and thus the reservoir storage reaches full at the end of stage *i* (Figure 3.1(c)) associated with a relatively small MU before the full state.

Taking all these three cases into consideration of the whole study horizon (N stages), the MU can be plotted by horizontal pieces linked by either an increasing or a decreasing piece with a slope. Each horizontal piece corresponds to a number of consecutive stages where the reservoir is usually neither full nor empty but the starting or ending point of each horizontal piece corresponds to either an empty or a full storage states (except for the first piece which starts from the given initial storage and the last piece which ends with the ending storage). Each increasing piece (with a nonlinear monotonically increase trend) corresponds to a number of consecutive stages with full ending storage, and each decreasing piece (with a nonlinear monotonically decrease trend) corresponds to a number of consecutive stages with empty ending storage. Specially, if two horizontal pieces are linked by only one stage with either full or empty ending storage, the increasing piece or decreasing piece becomes vertical. In summary, the reservoir storage states (including the empty/full cycles) correspond to piece-wise curves of MU, which are linear except for the case of consecutive empty/full storage over a number of stages.

For each horizontal piece containing N_H stages ($i = i_{k+1}, i_{k+2}, \dots, i_{k+N_H}$), the marginal utilities at any two consecutive stages follow Eq. (3.10), which gives $N_{HP} - 1$

equations with N_H unknows, i.e., the release (r_i^*) during the N_H stages. The initial and ending storage of the reservoir on the horizontal piece, s_{P0} and s_{PN}, should take empty storage, full storage, initial storage (s_0) or ending storage (s_N), thus according to the mass balance on a horizontal piece, Eq. (3.19) could be obtained.

$$\sum_{i=i_{k+1}}^{i_{k+N_{HP}}} r_j = \sum_{i=i_{k+1}}^{i_{k+N_{HP}}} I_j + s_{P0} - s_{PN} \tag{3.19}$$

Combining the $N_{HP} - 1$ equations given by Eq. (3.10) and Eq. (3.19), we could obtain N_{HP} equations with N_{HP} unknowns, and solve for r_i^* on the piece. For any stage on an increasing piece, the initial and the ending storage of the stage are both full storage; thus, according to mass balance, the release should be identical to the inflow on those stages. Similarly, for any stage on a decreasing piece, the initial storage and empty storage of the stage are both empty storage; the release should be identical to the inflow on these stages too. Overall, the release decision over all N stages can be determined by the procedures described above.

Furthermore, when the storage state turns from a non-empty state to an empty state, the cumulative release curve must be tangent to the cumulative inflow curve (Figure 3.1(b)) (see the proof of this tangent relationship in Appendix D.) Similarly, when the storage state turns from a non-full state to a full state, the cumulative release curve should be tangent to the curve, as shown in Figure 3.1(c), if the cumulative inflow curve is moved downward by the storage capacity (Figure 3.1(c)).

Based on the three cases shown between two stages as discussed above, two special cases are further discussed in the following. Case 4: the non-negative storage constraint becomes binding for the first time after the initial storage, and then the storage capacity constraint becomes binding at a later stage; and vice versa for Case 5. These two cases cover multiple stages instead of two as used in Cases 1-3.

Case 4:

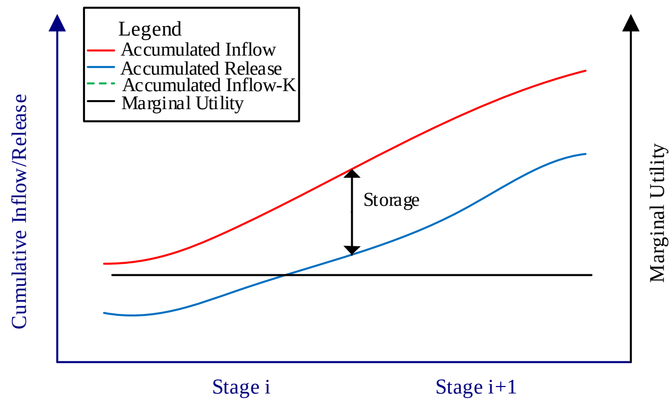
Under this case (Figure 3.1(d)), the non-negative storage constraint is binding first at stage *i* and then the storage capacity constraint is binding at a later stage *j*. Before stage *i*, the reservoir is neither empty nor full, and the MU relationships follow Eq. (3.10) (i.e., Case 1), i.e., the MU is identical from the initial stage up to stage *i*. At the end of stage *i*, the reservoir is empty, and possibly continues to be empty until stage *k,* for which Eq. (3.14) holds (i.e., Case 2). Along the curve connecting the two horizontal lines (horizontal line before stage *i* and line after stage *k*), the reservoir remains an empty state, the MU declines, and the reservoir releases all inflow at all stages from stage *i*+1 to *k*. Starting from stage *k*+1 and before stage *j*, Eq. (3.10) holds (i.e., Case 1) again as the reservoir is usually neither full nor empty, and Eq. (3.18) (i.e. Case 3) holds for stage *j* and the following stages with full ending storage, and the MU increases from a lower value to a higher value after each full state. Starting from next stage with non-full ending storage, Eq. (10) holds (i.e., Case 1) again, and so on. At a state of empty storage, the cumulative release curve is tangent to the cumulative inflow curve; while at a stage of full storage, the cumulative release curve is tangent to the curve that is the cumulative inflow curve moved downward by the storage capacity.

Define T_d as the period from the beginning to the time of the first occurrence of full (empty) storage and T_f as the period from the beginning to the time of the first occurrence of empty (full) storage (Figure 3.1(d), (e)), In this case, if T_d represents the period from the beginning to the first occurrence of empty storage and T_f as the period from the beginning to the first occurrence of full storage (Figure 3.1(d)). It is proved that as long as the inflow forecast in T_f is given, the optimal release decision in T_d will be determined no matter how inflow after T_f will be (See Appendix E). Actually, if the future inflow beyond T_f is abundant, it will be better to use more water within T_f and decrease the water storage at the end of T_f as much as possible. Under this situation, within T_d , the release decision will not be affected by the inflow beyond T_f, as the storage is empty at T_d and the water use within T_d already reaches the maximum. On the other hand, if there exists water stress in the future beyond T_f, more water should be left to the future and less water be used within T_f. However, at the end of T_f, if the storage constraint is binding, there is no more capacity to reserve water to the future. Thus, even if there is a drought event in the future, the optimal release decision within T_d will not change. Thus, under the condition of such empty-full states, forecast is not needed beyond T_f for the release decision within T_d.

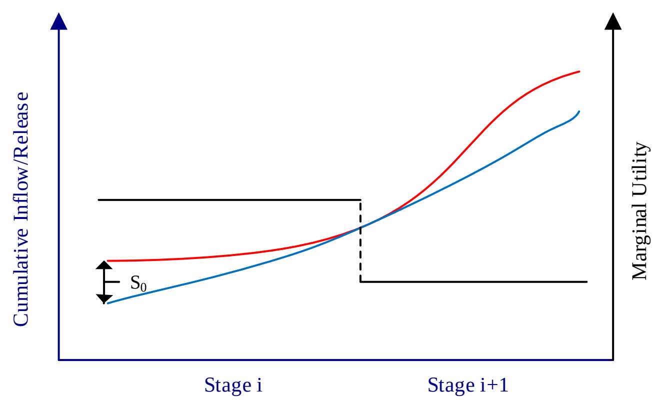
Case 5:

Under this case, the storage capacity constraint becomes binding first, followed by a binding non-negative storage constraint at a later stage. Assuming the storage capacity constraint becomes binding first at stage i and then the non-negative storage constraint becomes binding at a later stage j. Before stage i, the reservoir is usually neither empty nor full, the MU relationships follow Eq. (3.10) (i.e., Case 1), and the MU is identical before the end of stage i. At the end of stage i, the reservoir is full, and it possibly continues to be full until after stage k, where Eq. (3.18) (i.e., Case 3) holds. After each of the full storage states that can occur continuously between stage i and k, the MU increases continuously. The reservoir releases all inflow between stages i+1 to stage k. Starting from stage k+1 and before stage j, Eq. (3.10) holds (i.e., Case 1) again as the reservoir is usually neither full nor empty, and Eq. (3.14) (i.e. Case 2) holds for stage j and the following stages with empty ending storage, where the MU declines.

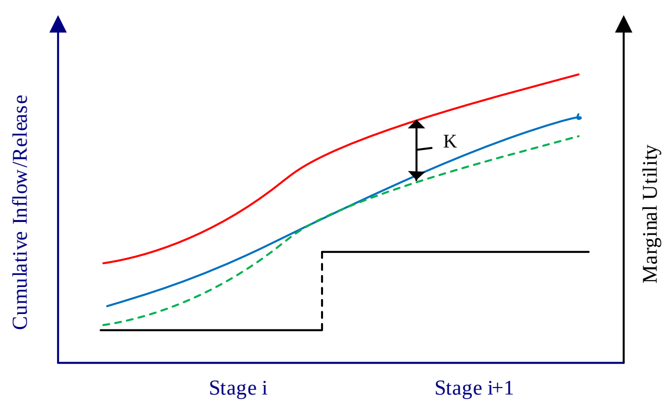
Under this case, T_d represents the period from the beginning to the first occurrence of full storage state and T_f as the period from the beginning to the first occurrence of empty storage (Figure 3.1(e)). As shown in Appendix E, similar to Case 4, it is proved T_f is the longest forecast horizon that provide useful information to the release decision within T_d. This can also be justified as follows: If future has a water deficit situation, less water should be used within T_f, and thus, the water storage at the end of T_f will be increased. Under this situation, only the release decision after T_d can be changed. This is because the storage capacity is binding at T_d, and the capacity of the reservoir limits the ability of the reservoir to store more water for the future. Under the water abundant situation, the release decision within T_f (thus also within T_d) will not be affected by the inflow beyond T_f, as the storage is empty at T_f, and, water use within T_f already reaches the maximum. Thus, under the condition of such full-empty states, forecast is not needed beyond T_f for the release decision within T_d.



(a) Case 1: with the binding mass balance constraint only



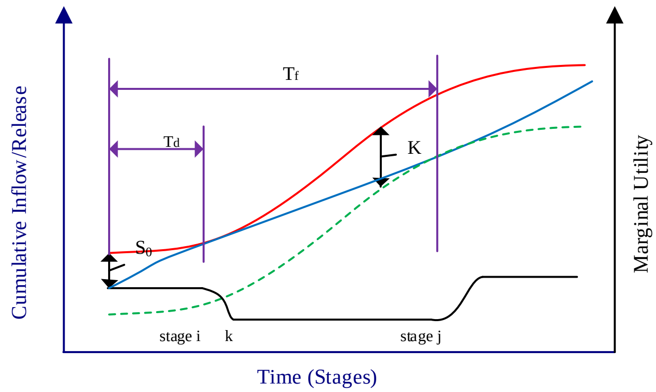
(b) Case 2: binding mass balance constraint with binding non-negative storage constraints



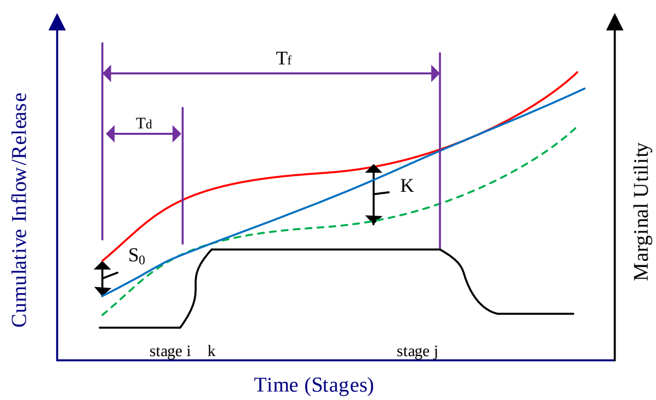
(c) Case 3: binding mass balance constraint with binding storage capacity constraints

Figure 3.1 Optimal reservoir release under the various cases ((a) to (e)).

Figure 3.1 (cont.)



(d) Case 4: the non-negative storage constraint binding first and then the storage capacity constraint binding (the increasing part and decreasing part of MU does not have a specific shape)



(e) Case 5: the storage capacity constraint binding first and then the non-negative storage constraint binding (the increasing part and decreasing part of MU does not have a specific shape)

After the first full-empty (empty-full) cycle, such empty-full or full-empty states can occur multiple times during the study period and the MU relationships and decision rules discussed above repeat, though it will no longer influence the release decision at the current stage.

3.3 Procedures to Determine Effective Forecast Horizon

The problem of determining the EFH is stated as follows: Given inflow forecast time series, $I_1, I_2, \ldots, I_N$, and corresponding inflow uncertainty, which is given as certain confidence intervals, such as $[\underline{I_1}, \overline{I_1}]$, $[\underline{I_2}, \overline{I_2}]$, … , $[\underline{I_N}, \overline{I_N}]$, the EFH will satisfy the following condition: the decision error at the current stage does not go beyond a given error bound, EB, i.e.,

$$\forall R_1^*, EB \geq |R_1 - R_1^*| \tag{3.20}$$

where, R_1 is the optimized release decision at the current stage under forecast with uncertainty; R_1^* is the optimal release decision under a perfect forecast, which is usually unknown given that a perfect forecast does not exist for decision making. EB is the prescribed error bound.

To determine the EFH, a longest forecast horizon (LFH) is first proposed that provides useful information for the current stage decision. You and Cai (2008) defined the forecast horizon as forecast lead time which would enable the optimal release decision in the decision horizon without any inflow forecast beyond the forecast horizon. Zhao et al. (2012) showed the existence of such a forecast horizon under imperfect forecast using numerical experiments. Similarly, we show that there exists a longest forecast horizon (LFH), beyond which the future inflow information and associated uncertainty will no longer affect the error involved in the release decision at the current stage, i.e., all forecast lead time greater or equal to the LFH will result in the same error bound of release decision at current stage (Figure 3.2). Thus, the LFH will provide the longest possible forecast lead time as a candidate for the EFH. The LFH is similar as the forecast horizon as discussed in You and Cai (2008) but characterized by error involved in release decision as shown in Figure 3.2. As discussed later, if the LFH satisfies the criterion for EFH, then the EFH should be identical to the LFH; otherwise, the EFH should be shorter than the LFH and it is conditioned with a given storage at the end of the EFH (i.e., the ending storage).

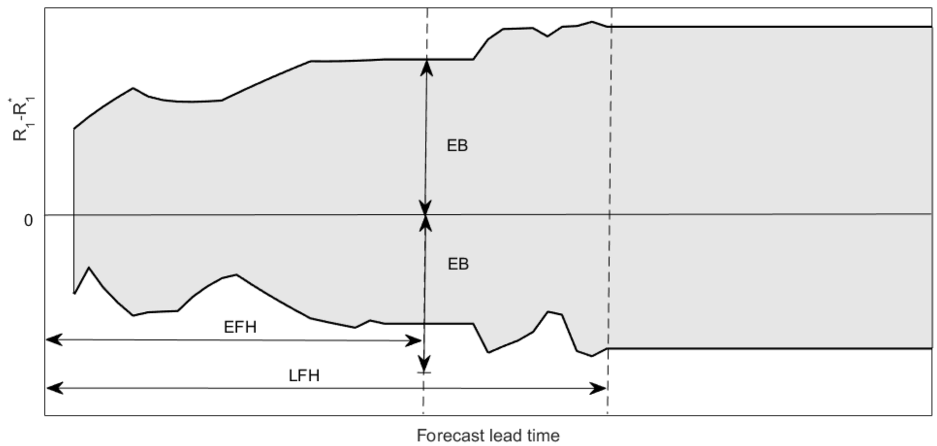


Figure 3.2 Illustration of the EFH and LFH

Assume a concave utility function, $b(r)$, $b'(r) \to 0$, when $r \to \infty$; $b'(r) \to \infty$, when $r \to 0$. For a given time period, under the most optimistic inflow conditions beyond the time period, i.e., $r \to \infty$, $b'(r) \to 0$ beyond the time period, then the marginal utility will decrease at the end of the time period (as the MU approaches 0 after the time period), thus the ending storage should be empty; under the most pessimistic inflow conditions beyond the time period, i.e., $r \to 0$, $b'(r) \to \infty$ beyond the time period, then the marginal utility will increase at the end of the time period (as the MU approaches infinity after the time period), thus the ending storage should be full. Thus, if the release decision at current stage is irrelevant to the ending storage specified at the end of the forecast horizon, the release decision is also irrelevant to the inflow condition beyond the forecast horizon. Thus, the LFH also represents the shortest forecast lead time which would lead to a release decision at the current stage with the release error bound irrelevant to the given storage at the end of the forecast lead time. As shown in Appendix F, assuming that actual inflows stay within the forecasted range under a given confidence level, with any given ending storage, S_N, as constraints, the lower bound and the upper bound of the release at the current stage will be specified by a the most pessimistic and most optimistic inflow scenario. With the most pessimistic forecast, the cumulative inflow at each stage is lower than the cumulative inflow over all scenarios; with the most optimistic forecast, the cumulative inflow at each stage is higher than that over all scenarios. Thus, if the most pessimistic and most optimistic inflow scenario provide consistent upper bound and lower bound of the optimal release decision at current stage with any given ending storage, then the lower bound and upper bound of the optimal release will be irrelevant to the ending storage, which also means that, with given release decision at the current stage made by some reservoir operation policy, the release error bound at current stage will be irrelevant to the ending storage.

Actually, as shown in Section 3.2 with Case 4 and Case 5, for any inflow scenario, if there exists full-empty or empty-full cycle in the optimal release decision, then the decision at the current stage is irrelevant to the forecast beyond the cycle. Thus, if both the most pessimistic and optimistic inflow scenarios include sufficient variability, i.e., including a full-empty or empty-full cycle in the corresponding optimal release decision, there will exist a LFH.

Thus, two special scenarios, i.e. the most optimistic one or the most pessimistic one, will be synthesized based on all possible inflow scenarios to determine the LFH. As proved in Appendix G, for any inflow scenario, if the release decisions at the current stage resulting from an empty ending storage constraint or a full ending storage constraint are identical, then the release decisions at the current stage are irrelevant to the ending storage. The property holds for the most optimistic or the most pessimistic scenario, and thus with inflow forecast with in LFH,

$$R^*_{1,opt,f} = R^*_{1,opt,e} \tag{3.21}$$

$$R^*_{1,pes,f} = R^*_{1,pes,e} \tag{3.22}$$

where $R^*_{1,opt,f}$ / $R^*_{1,opt,e}$ are the release decision at the current stage under the most optimistic forecast with full/empty ending storage constraint; $R^*_{1,pes,f}/R^*_{1,pes,e}$ are the release decisions under the most pessimistic forecast. Over all the tested LFH, the shortest one is selected as the LFH.

Further, the following procedures are proposed to determine the LFH (See the flowchart shown in Figure 3.3):

(a) Prepare the following input data set: inflow forecast for future N stages, $I_1, I_2, \dots, I_N$; inflow forecast under the most optimistic scenario, $I_{1,opt}, I_{2,opt}, \dots, I_{N,opt}$ and the most pessimistic scenario, $I_{1,pes}, I_{2,pes}, \dots, I_{N,pes}$ (following the definition given before); demand for each stage, $D_1, D_2, \dots, D_N$; initial storage at the beginning, S_0; storage capacity of the reservoir, *K*; and utility functions for each stage, $b_i(\cdot)$.

(b) Set the initial candidate of LFH as the study horizon (N) (the longest LFH) and examine whether the $LFH = N$ satisfies the requirement for LFH, i.e. Eq. (3.21) & (3.22). If not, then, the LFH should be greater than the study period (N) and the program stops. Otherwise, $N - 1$ will be set as the candidate for next test. If N– 1 does not satisfy the requirement for LFH, then, N remain as the LFH; otherwise, N– 2 is set as next candidate, and so on. When $N - k$ does not stratify the requirement, then $N - k + 1$ is the final LFH.

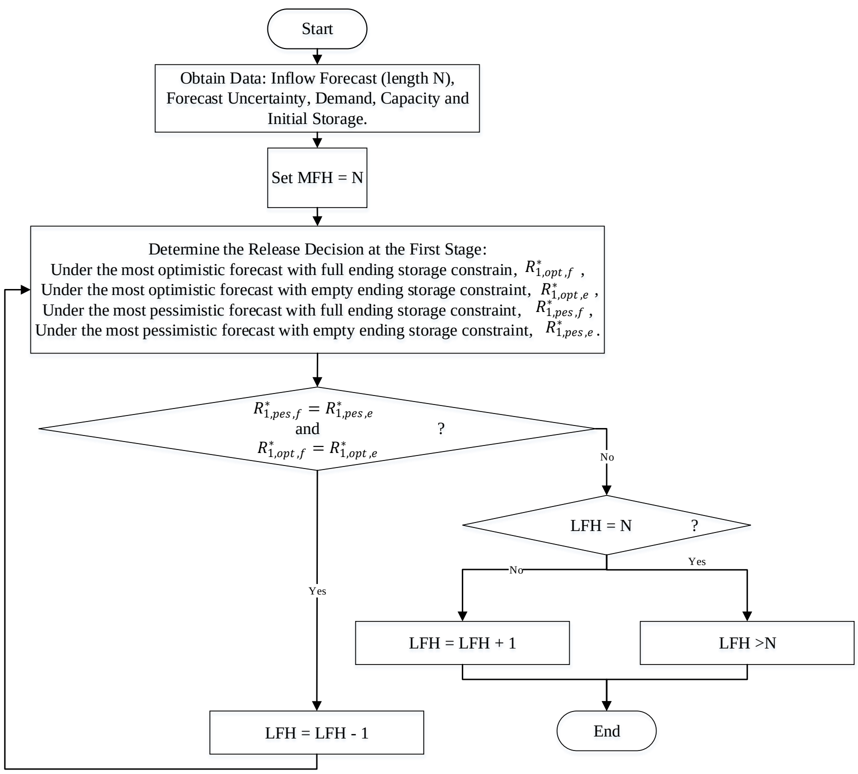


Figure 3.3 Procedure for Determining the LFH

After the LFH is determined, the following procedures are applied to further determine EFH. First, we specify the criterion for EFH. With given inflow forecast for future N stages, $I_1, I_2, \ldots, I_N$, and corresponding inflow uncertainty under certain confidence levels, such as $[\underline{I_1}, \overline{I_1}]$, $[\underline{I_2}, \overline{I_2}], \ldots, [\underline{I_N}, \overline{I_N}]$, a release decision for the current stage, R_1, could be made following some reservoir operation policy. With a given error bound, EB, the requirement for the release decision at the current stage is given by Eq. (3.20). If an estimate of the unknown actual optimal release decision, R^*_1, could be made, then, Eq. (3.20) could be used to examine whether the release decision satisfy the error bound. Assuming that under a given uncertainty level, the actual cumulative inflow will stay within a range, i.e.,

$$CI_i^* \subset \left[\underline{CI_i}, \overline{CI_i}\right], \forall i \quad (3.23),$$

where, CI_i^* is the unknown actual cumulative inflow at stage i. Based on the properties of the optimal release decision from the multi-stage optimization model as discussed above, it is proved that among all possible inflow scenarios, the most optimistic scenario together with the lower bound of the ending storage, $\underline{S_N}$ (prescribed), will provide the largest amount of release, $\bar{R}_1^*$, at current stage, and the most pessimistic scenario together with the upper bound of ending storage, $\overline{S_N}$, will provide the smallest amount of release, $\underline{R_1^*}$, at current stage (See Appendix F for details.) Thus $\underline{R_1^*}$ and $\bar{R}_1^*$ provide a range of the optimal release, i.e.,

$$\underline{R_1^*} \leq R_1^* \leq \bar{R}_1^* \quad (3.24)$$

$$R_1 - \bar{R}_1^* \leq R_1 - R_1^* \leq R_1 - \underline{R_1^*} \quad (3.25)$$

$$|R_1 - R_1^*| \leq \max\left(|R_1 - \bar{R}_1^*|, \left|R_1 - \underline{R_1^*}\right|\right) \quad (3.26)$$

thus, if

$$\max\left(|R_1 - \bar{R}_1^*|, \left|R_1 - \underline{R_1^*}\right|\right) \leq EB \quad (3.27)$$

then, the attempted forecast horizon is a candidate for the EFH.

Based on this additional criterion (Eq. 3.27), the following procedure is proposed to determine the EFH based on the LFH (See the flowchart in Figure 3.4).

(a) Set the error bound EB together with LFH that is determined via the procedures described above.

(b) If the LFH is greater than N, then set the initial EFH as N; otherwise, set the initial EFH, EFH_0, as the determined LFH.

(c) If $EFH_0 = N,$ obtain the estimate of ending storage at the end of the study horizon (which is needed for this condition), and then users can determine release decision at current stage (R_1) using forecast information within the study horizon and their decision-making policies; if $EFH_0 = LFH$, then R_1 can be determined without the ending storage specification.

(d) Examine whether the current candidate EFH_0 satisfies the given EB (Eq. (3.27)) by determining $\bar{R}_1^*$ and $\underline{R}_1^*$ as defined above. Note when the candidate EFH_0 is found to be less than *LFH,* then an estimated ending storage range (prescribed) is to be used as constraints. The detailed procedures to estimate $\bar{R}_1^*$ and $\underline{R}_1^*$ are given later as shown in Figure 3.5.

(e) If the EFH candidate satisfies the given EB, then stop with a determined EFH; otherwise, the current EFH candidate will be shortened by one period and the procedures go back to (d).

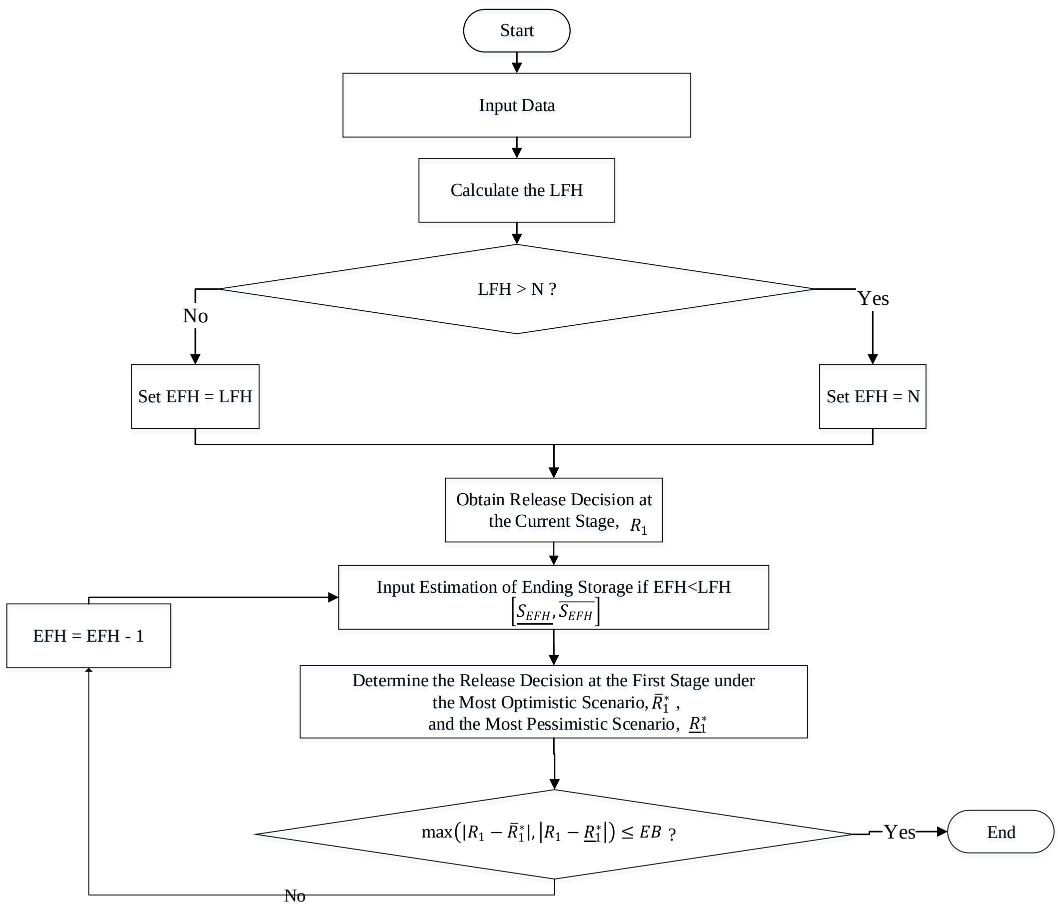


Figure 3.4. Flowchart for determining the EFH

Following the discussion above, to complete the procedures to determine *EFH*, we need to determine $\bar{R}_1^*$ and $\underline{R}_1^*$, which are optimal release decisions under the most optimistic and pessimistic inflow forecasts respectively. For this purpose, the procedures to solve the optimal release decision at current stage under given inflow scenario are presented as below

(Figure 3.5), which is based on the tangent relationship between the cumulative inflow curve and the cumulative release curve at a certain time point where the MU switches from high to low value (when the storage is empty) or from low to high value (when the storage is empty), characterized by storage capacity limit, inflow conditions, and current and future water demand, as discussed above, i.e.,

(1) Starting from the first stage (i=1), calculate the range of the MU$[\underline{\lambda}_i, \bar{\lambda}_i]$, where $\underline{\lambda}_i$ refers the empty storage at the end of stage i, and $\bar{\lambda}_i$ to the full storage at the end of stage i. For both cases, the MU is identical before stage i, i.e., $\underline{\lambda}_i$ is the solution of Eq. (3.28) & Eq. (3.29), and $\bar{\lambda}_i$ is the solution of Eq. (3.30) & Eq. (3.31),

$$b_1'(r_1) = b_2'(r_2) = \cdots = b_i'(r_i^*) = \underline{\lambda}_i \tag{3.28}$$

$$\sum_{j=1}^{i} r_j = \sum_{j=1}^{i} I_j + s_0 \tag{3.29}$$

$$b_1'(r_1) = b_2'(r_2) = \cdots = b_i'(r_i^*) = \bar{\lambda}_i \tag{3.30}$$

$$\sum_{j=1}^{i} r_j = \sum_{j=1}^{i} I_j + s_0 - K \tag{3.31}$$

For the last stage N, there is a single value for λ with a given ending storage s_N, as shown in Eq. (3.32) & Eq. (3.33).

$$b_1'(r_1) = b_2'(r_2) = \cdots = b_i'(r_N^*) = \underline{\lambda}_N = \bar{\lambda}_N \tag{3.32}$$

$$\sum_{j=1}^{i} r_j = \sum_{j=1}^{i} I_j + s_0 - s_N \tag{3.33}$$

(2) Starting from the first stage, identify the intersection of the ranges of the MU before stage i, $\cap_{k=1}^{i}[\underline{\lambda}_k, \bar{\lambda}_k]$. Corresponding to any pair of $[\underline{\lambda}_k, \bar{\lambda}_k]$, the reservoir storage falls in a feasible range (i.e., between 0 and storage capacity) at stage $k = 1,2, \ldots, i$. Thus $\cap_{k=1}^{i}[\underline{\lambda}_k, \bar{\lambda}_k]$ represents the range of MU that can be achieved at stage i without violating the capacity and non-negativity constraint before stage i.

Furthermore, at stage i, if

$$\bigcap_{\mathrm{k=1}}^{\mathrm{i}}[\underline{\lambda}_k, \bar{\lambda}_k] = \Phi \tag{3.34}$$

which means that there does not exist any single MU value over all stages before i and thus either non-negative storage or storage capacity constraint is binding between the beginning and stage i.

Thus, one of the following cases will exist:

(a) If $\underline{\lambda}_i > \min\{\bar{\lambda}_1, \bar{\lambda}_2, \dots, \bar{\lambda}_{i-1}\}$, then there is a full state before stage i, and following the tangent relationship mentioned above, the full state occurs at the end of the stage i_f, $\bar{\lambda}_{i_f} = \min\{\bar{\lambda}_1, \bar{\lambda}_2, \dots, \bar{\lambda}_{i-1}\}$. Then, the optimal release decision at the current stage can be calculated using the following equations.

$$b_1'(r_1) = b_2'(r_2) = \cdots = b_{i_f}'\left(r_{i_f}^*\right) \tag{3.35}$$

$$\sum_{j=1}^{i_f} r_j = \sum_{j=1}^{i_f} I_j + s_0 - K \tag{3.36}$$

(b) If $\bar{\lambda}_i < \max\left\{\underline{\lambda_1}, \underline{\lambda_2}, \dots, \underline{\lambda_{i-1}}\right\}$, then there exists an empty state before stage i, and according to the tangent relationship, the empty state occurs at the end of the stage i_e, $\underline{\lambda_{i_e}} = \max\left\{\underline{\lambda_1}, \underline{\lambda_2}, \dots, \underline{\lambda_{i-1}}\right\}$. Then, the optimal release decision at the first stage can be calculated using the following equations.

$$b_1'(r_1) = b_2'(r_2) = \cdots = b_{i_f}'\left(r_{i_e}^*\right) \tag{3.37}$$

$$\sum_{j=1}^{i_e} r_j = \sum_{j=1}^{i_e} I_j + s_0 \tag{3.38}$$

(c) Otherwise, if Eq. (3.34) is not satisfied at all stages, a single MU could be obtained at all stages marinating all constraints. Then, the optimal release decision at the current stage can be calculated using the following equations.

$$b_1'(r_1) = b_2'(r_2) = \cdots = b_N'(r_N^*) \tag{3.39}$$

$$\sum_{j=1}^{N} r_j = \sum_{j=1}^{N} I_j + s_0 - s_N \tag{3.40}$$

The procedures discussed above are given in Figure 3.5.

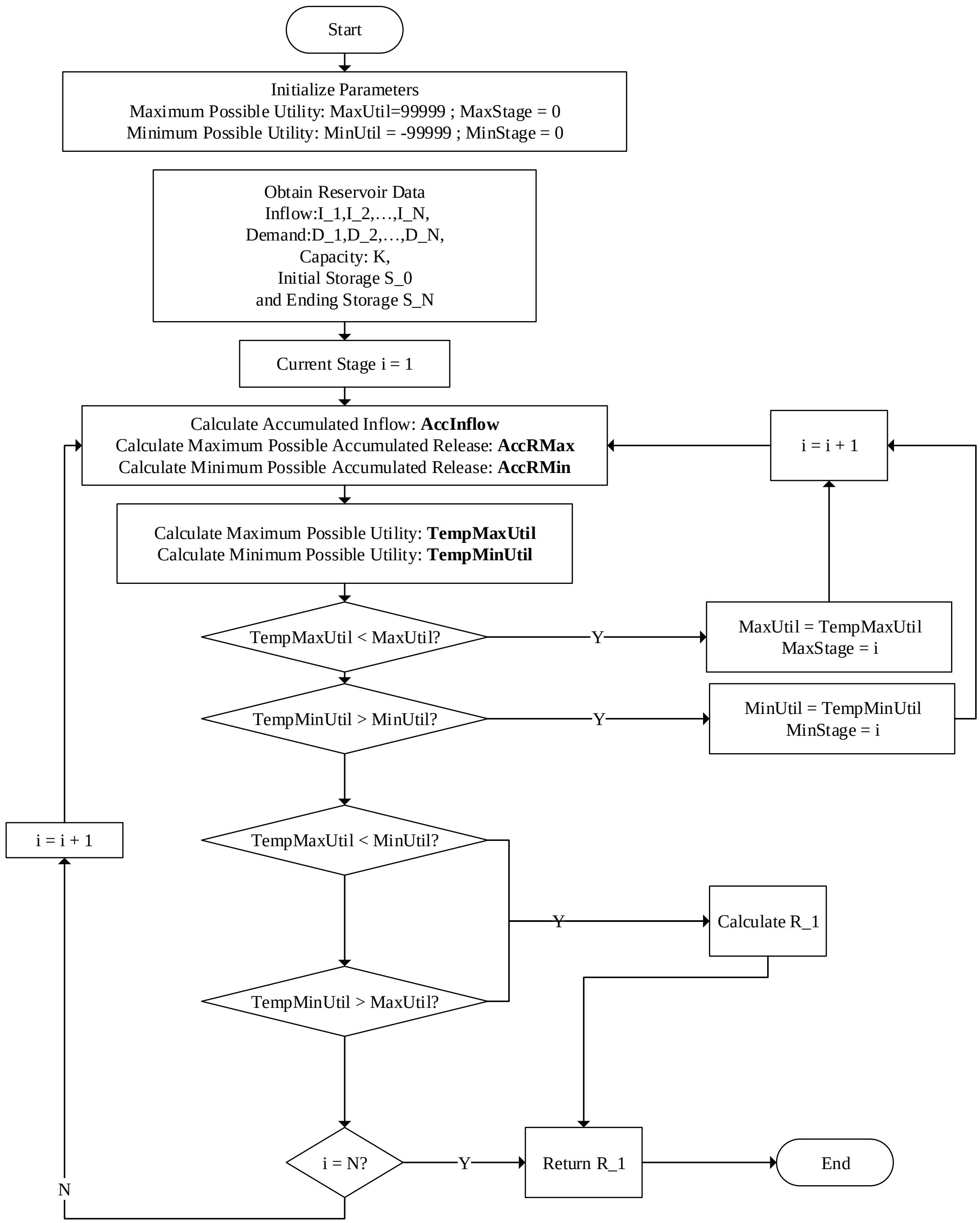


Figure 3.5 Flowchart for determining release decision at current stage

3.4 A Synthetic Case Study

A synthetic case study is conducted by generating inflow time series using the Thomas-Fiering model (Thomas & Fiering, 1962),

$$q_{i+1} = \mu + \rho_{flow}(q_i - \mu) + \sqrt{1 - \rho_{flow}^2}(\mu C_v)\omega \tag{3.41}$$

where q_i is the inflow at stage i, μ is the average inflow, which is also set as the initial inflow at the first stage, ρ_{flow} represents the temporal correlation of the inflow, and C_v represents the inflow variability, and ω is a random number generated from standard normal distribution.

Following Zhao et al. (2012), the forecast error is characterized by a distribution with zero mean and a variance linearly increasing with the forecast lead time but constrained by an upper bound of the streamflow variability,

$$\zeta_i^2 = \min(i\sigma^2, \mu^2 C_v) \tag{3.42}$$

where, ζ_i^2 is the variance of forecast error at stage i; σ^2 is the variance of forecast error at the first stage.

Assuming the forecast error is irrelevant over stages, and the inflow for the most optimistic scenario and the most pessimistic scenario can be estimated by

$$q_{opt,i} = q_i + 2\zeta_i \tag{3.43}$$

$$q_{pes,i} = q_i - 2\zeta_i \tag{3.44}$$

where, $q_{opt,i}$ is the estimate of the inflow at stage i of the most optimistic scenario, and $q_{pes,i}$ is that of the most pessimistic scenario.

The MU function $b'(\cdot)$ is assumed to be a function of release/demand ratio with the same form at all stages, i.e.,

$$b_i{}'(r_i) = b'\left(\frac{r_i}{d_i}\right) \tag{3.45}$$

where, r_i is the release at stage i, d_i is the demand at stage i.

The estimate of ending storage is given by$[S_{i,opt} + \epsilon_i - dS, S_{i,opt} + \epsilon_i + dS]$, where $S_{i,opt}$ is the actual optimal decision, ϵ_i is a bias term given by a Normal distribution, dS represents the estimation accuracy, which is constant over stages as it is assumed that this estimation is obtained from existing information other than the forecast.

The demand is set as a periodic piece-wise constant function, consisting of low demand periods and high demand periods; each period consists of 6 stages, starting from a low demand period. The demand at each stage is set as 1 and $1 + \alpha$, during the low and high demand period, respectively.

The longest available forecast length is set as $N = 60$.

Six experiments are conducted to explore the sensitivity of the EFH and the LFH to different parameters, as summarized in Table 1. In each experiment, one parameter is adjusted with other parameters set as default values. For some combinations of parameters, there might not exist a determined EFH because the required EB is too strict; or there might not exist a determined LFH as the given maximum available forecast length (N) is not long enough.

Table 3.1 Default values for parameters and range of adjusted parameters in each experiment

Experiments	Parameters	Default values	Value ranges
1	K	3	1-8
2	C_v	0.3	0.1-1
3	σ	0.04	0.001-0.1
4	α	1	1-3
5	dS	0.5	0-1
6	EB	0.15	0.1-0.25

The experiment results are shown in Figure 3.6.

Figure 3.6(a) shows the relationship between storage capacity, K, and EFH and LFH. When the storage capacity is small, the ability of the reservoir to regulate inflow for water supply in a long period is weak, and thus, the LFH is short. A short LFH usually goes with small cumulative uncertainty which makes EFH identical to LFH with a high likelihood to satisfy the prescribed EB. When the storage capacity increases, the ability of the reservoir to regulate inflow to satisfy water supply in long term period becomes stronger, and the LFH increases, which likely results in larger cumulative uncertainty within the LFH, and thus the EFH becomes shorter than the LFH to satisfy the requirement of EB, as shown in Figure 3.6(a) with larger capacity values.

Figure 3.6(b) shows relationship between the inflow variation, C_v, and EFH and LFH. There is a slightly negative correlation between the inflow variation and the LFH, as a result of decreasing regulation ability in a long period with increasing inflow variation.

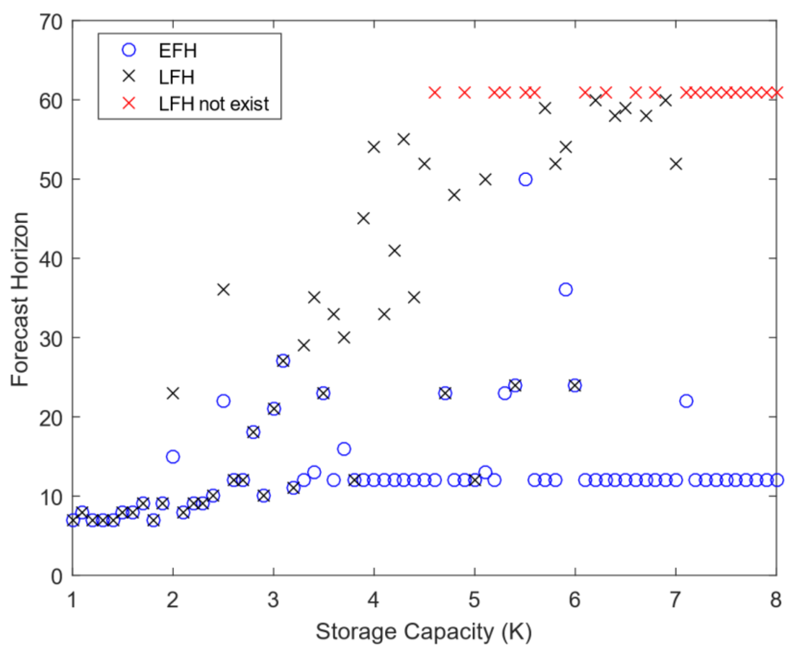
When the inflow variation is large (equivalent the case that the reservoir capacity is small), the LFH is small, and consequently there is higher chance for the EFH identical to the LFH, as the cumulative uncertainty at the LFH might be small enough to satisfy the given EB. When the inflow variation is small, the LFH is large, and consequently, the cumulative uncertainty at the LFH is too large to satisfy the given EB, thus, the EFH is shorter than the LFH and depends on the estimates of the ending storage. However, as shown in Figure 3.6(b), when the inflow variation is small (i.e., most stable inflow), a determined EFH rarely exists with the given EB and ending storage. This can be explained as below: more stable inflow corresponds to a longer LFH, which is likely associated with higher forecast uncertainty and thus a larger EB is needed to make the error involved in release decision satisfy the EB.

Figure 3.6(c) shows the relationship between the forecast uncertainty level,σ, and EFH and LFH. With increasing forecast uncertainty level, the EFH decreases quickly, and no determined EFH exists when the forecast uncertainly level is high, which is a direct result of increasing uncertainty. The relationship between the LFH and the forecast uncertainty level, σ, also depends on how uncertainty changes with the forecast lead time. If the uncertainty level increases rapidly with the forecast lead time, then a shorter LFH will be preferred. In Figure 3.6(c), when the uncertainty level, σ, is relatively small, increasing σ makes the uncertainty increases faster with forecast lead time, thus, will decreases the LFH, i.e., there will be a preference on short term forecast with less uncertainty. But due to the fact that we limit the forecast uncertainty no larger than the inflow variation when the uncertainty is large, when the uncertainty level, σ, is large, the uncertainty will become stable soon after the first few stages, and, there will be no longer a preference for short LFH.

Figure 3.6(d) shows the relationship between demand variation, α, and EFH and LFH. Increasing the fluctuation of demand is similar to increasing the variation of inflow. Thus, in general, the LFH decreases with increasing demand variation, and thus the chance for the EFH to be identical to the LFH will be higher as the cumulative uncertainty within the LFH becomes smaller. This figure also shows a slightly positive relationship between α and the EFH, which is caused by setting that the first six stages as a low demand period in the case study, i.e., the increasing of α will decrease the absolute value of the release at the first stage, and thus, make it easier to satisfy the given EB, which is an absolute error.

Figure 6(e) shows the relationship between the uncertainty of the ending storage estimate, dS, and EFH and LFH. By definition, the LFH is irrelevant to the ending storage estimate. As shown in the figure, a determined EFH exists with a higher possibility with a smaller ending storage estimation uncertainty, as it will allow higher inflow uncertainty with a given error bound. However, the benefit of decreasing the ending storage estimation uncertainty is not apparent, as the cumulated inflow uncertainty is dominant unless the inflow uncertainty is very small.

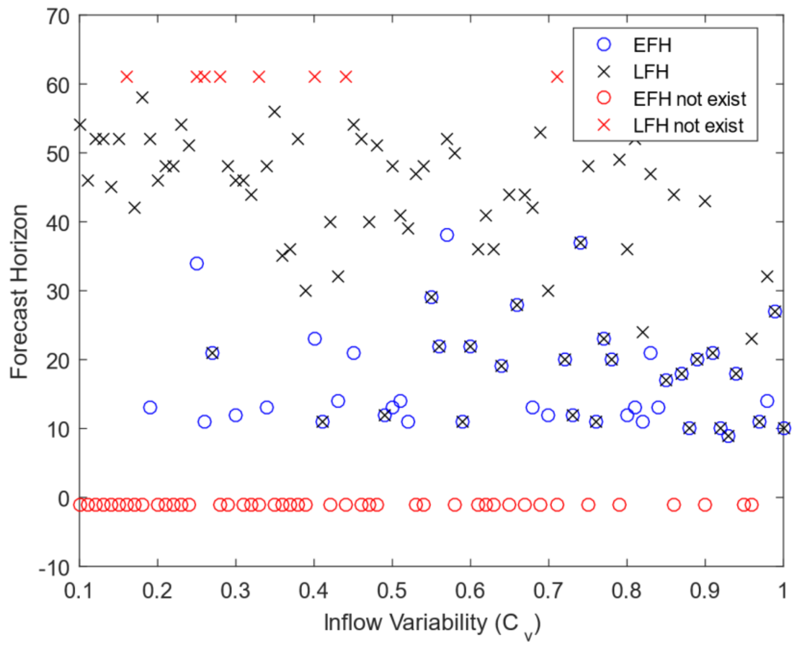
Finally, Figure 6(f) shows the relationship between EB and EFH and LFH. By definition, the LFH is irrelevant to a given error bound. It is clear that a larger EB will lead to a larger EFH as it will allow higher inflow uncertainty.



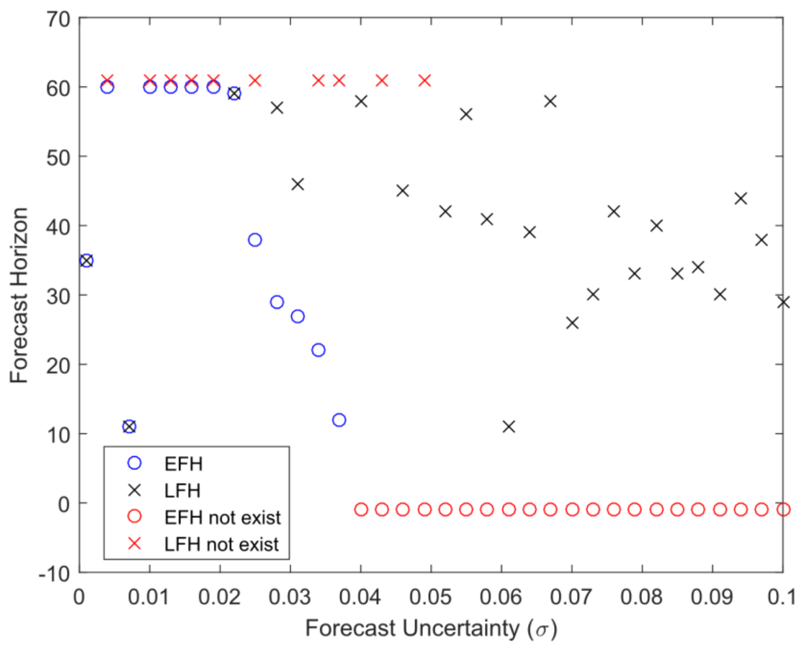
(a) Experiment 1: relationship between K and the EFH and the LFH

Figure 3.6 Results of six experiments

Figure 3.6 (cont.)

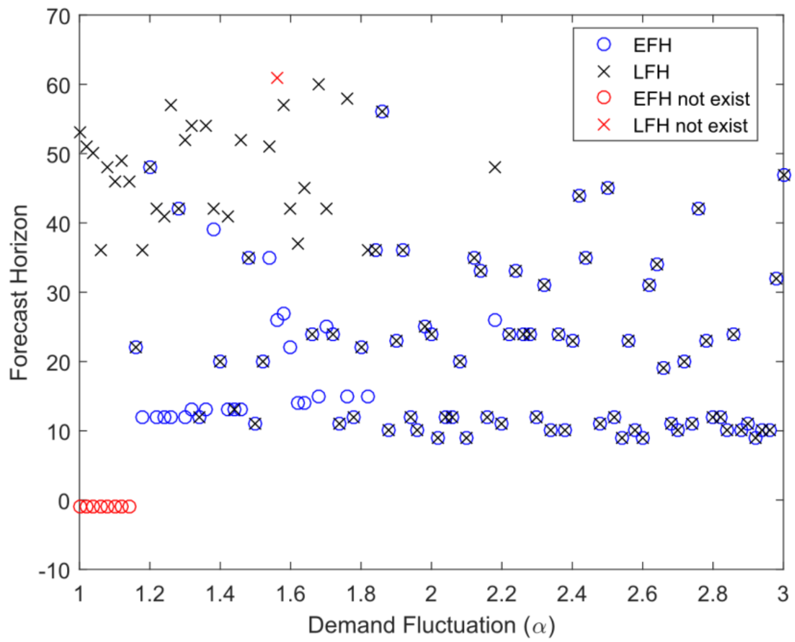


(b) Experiment 2: relationship between C_v and the EFH and the LFH

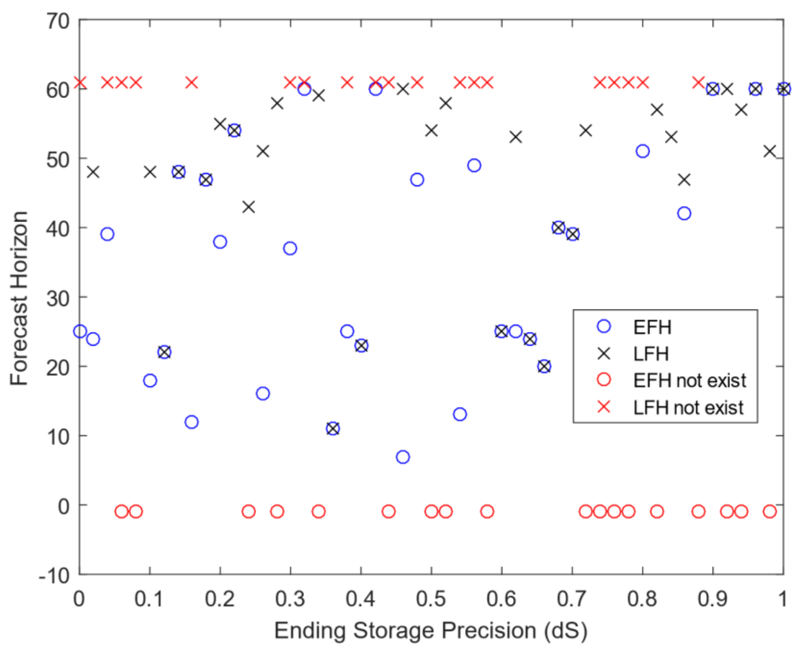


(c) Experiment 3: relationship between σ and the EFH and the LFH

Figure 3.6 (cont.)

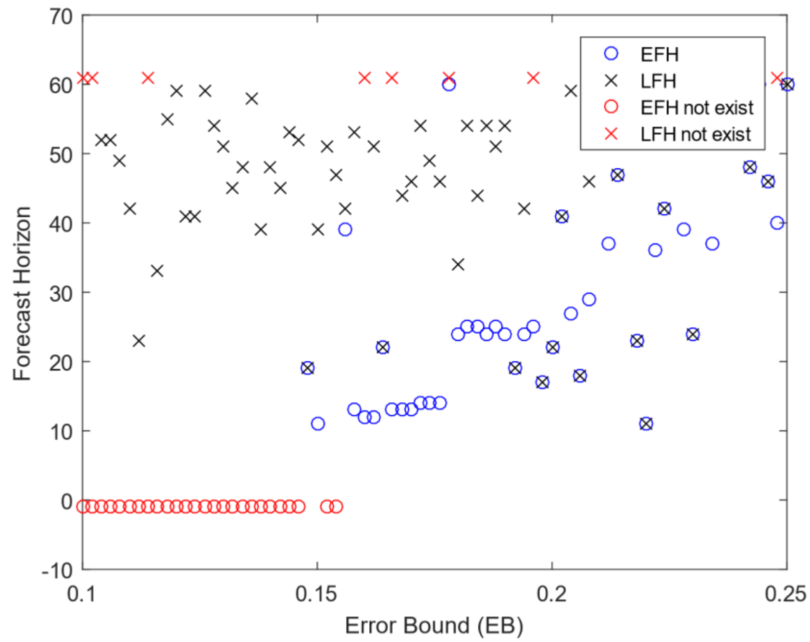


(d) Experiment 4: relationship between α and the EFH and the LFH



(e) Experiment 5: relationship between dS and the EFH and the LFH

Figure 3.6 (cont.)



(f) 6: relationship between EB and the EFH and the LFH

3.5 Conclusions

This study continues the efforts to determine the effective forecast horizon. A multi-stage reservoir operation model is first set up for a single reservoir with single a demand site, and the properties of the optimal solution, e.g. the relationships of MU and the tangent relationship between the cumulative release curve and the cumulative inflow curve, are analyzed, and the influence of forecast uncertainty on release decision is discussed. For a given cumulative inflow forecast, $I_1, I_2, \dots, I_N$, characterized with inflow uncertainty time series under certain confidence levels, such as $[\underline{I_1}, \overline{I_1}]$, $[\underline{I_2}, \overline{I_2}]$, …, $[\underline{I_N}, \overline{I_N}]$, and the estimate of ending storage, $\left[\underline{S_N}, \overline{S_N}\right]$, the most optimistic scenario with the lower bound of ending storage, $\underline{S_N}$, will provide the largest amount of release, $\bar{R}_1^*$, at current stage. Meanwhile, the most pessimistic scenario with the upper bound of ending storage, $\overline{S_N}$ will provide the smallest amount of release,$\underline{R_1^*}$, at the current stage.

To provide an upper bound for the EFH candidates, the concept of the longest forecast horizon (LFH), beyond which the future inflow information and associated uncertainty will no longer affect the error involved in the release decision at the current stage, is proposed. Correspondingly, the criteria for determining the LFH and the EFH are further developed. If the LFH satisfies the criterion for EFH, then the EFH is identical to the LFH; otherwise, the EFH is shorter than the LFH and it is conditioned with a given storage at the end of the EFH. Based on these criteria, procedures for determining the EFH are proposed.

Finally, a case study is conducted with synthesized inflow time series. The case study shows how EFH and LFH are affected by the storage capacity, the inflow variation, the forecast uncertainty level, the demand variation, the uncertainty of the ending storage estimates, and the given error bound. It is shown that the LFH has a positive relationship with storage capacity, negative relationship with inflow variation and demand variation, and no obvious relationship with the uncertainty of the ending storage estimates and the given error bound. The relationship between the LFH and the forecast uncertainty level depends on the characteristics of the inflow forecast uncertainty. Results show that the EFH has slight negative relationship with the uncertainty of the ending storage estimation, strong negative relationship with the forecast uncertainty level and strong positive relationship with the given error bound, and the EFH is more probable to exist under conditions where a short LFH exist.

CHAPTER 4: DECISION SUPPORT TOOL DEVELOPMENT

A prototype decision support tool for reservoir operation based on analytically derived reservoir operation rules (or empirical rules) is developed in this section. The prototype tool provides a web interface for users to upload data, select operation rules, and view results. This work in this chapter shows how to apply analytically derived reservoir operation rules to guide real-world reservoir operation.

4.1 System Design

This prototype decision support system provides the following functions, including those for authenticating system, defining reservoirs, uploading data, selecting analytical reservoir operation rules from the database, and displaying results. The detailed description of each function is described as follows:

(1) Authentication: users need to login to the system to set up reservoirs, manage reservoir data, select rules, and view results.

(2) Reservoir definition: the system assigns a unique ID for the reservoir, which will be used to identify the reservoir in the database system to link the properties, data, and the results to the reservoir. Then, the system will require some key properties of the reservoir (system) from users as inputs, e.g., reservoir system type (single reservoir, system of reservoirs in parallel, system of reservoirs in cascade), reservoir purpose (water supply, energy production, flood control, etc.)

(3) Date upload: upload the data of the reservoir (system) to the decision support system. The system is designed to handle both single values and time series data. Single value data, such as the number of reservoirs in the system, the number of stages, is input directly from the interface and saved in the database, and time series data, such as the demands over stages, the inflow over stages, is uploaded as files, and the database system is used to store the location of the file.

(4) Rule selection: the system provides users a list of analytical rules matching the properties of the reservoir (system), and provides a list of necessary data required by each rule. For example, for a single reservoir for water supply purpose, the system will match a list of reservoir operation rules, such as calculating the optimal release decisions, providing the effective forecast horizon (EFH) and longest forecast horizon (LFH), etc. Users can select a rule from the system to be applied to the reservoir after necessary data being uploaded. Once an analytical rule is selected, the system will prepare input files for the corresponding program of the rule, and execute the program.

(6) Result display: the results by each of the programs will be displayed via figures, tables, and text.

Figure 4.1 provides a flow chart for using the prototype decision support system, showing the procedures to use the functions described above.

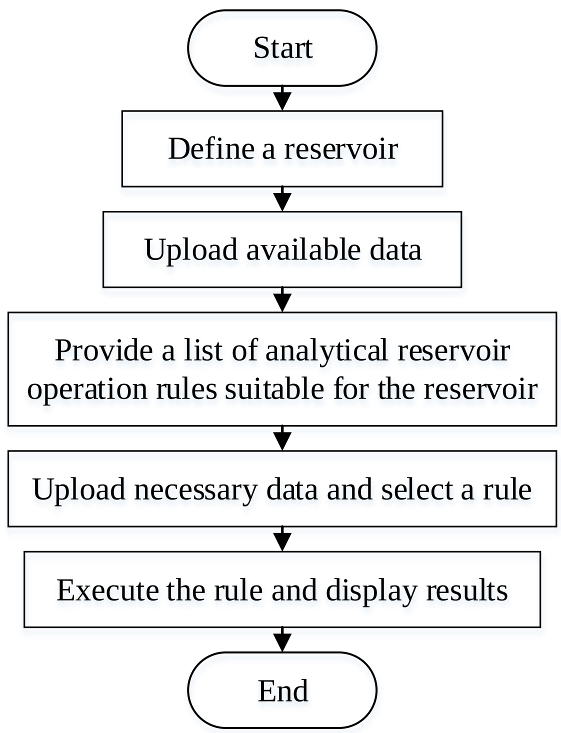


Figure 4.1 Flow chart for applying an analytical rule

4.2 An Illustrative Case

In this section, an illustrative case is provided to show the procedures of using the prototype decision support system. As an example, the optimal release decisions for a single water supply reservoir are provided based on the analytical results, i.e., the reservoir is empty when the marginal utility (MU) decreases, and full when the MU increases; and, otherwise, the MU is constant over stages.

First, we login the system, and the web-based homepage is shown as Figure 4.2.

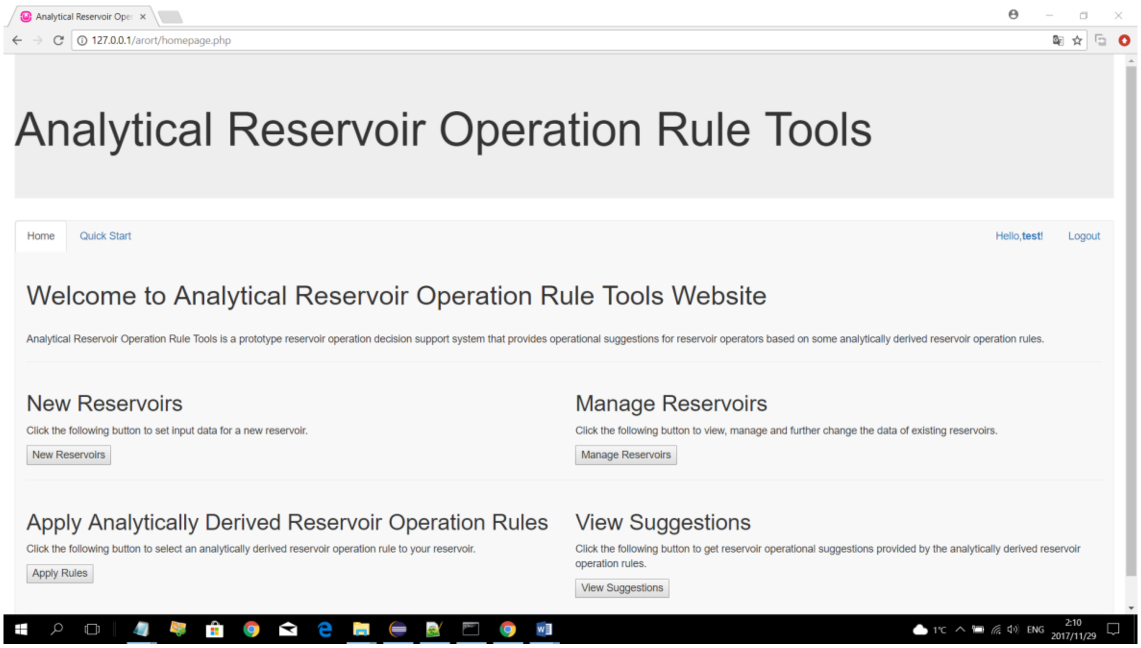


Figure 4.2 The webpage of the decision support tool

Then, a single reservoir system for water supply purpose is set up as shown in Figure 4.3 and data is uploaded as shown in Figure 4.4.

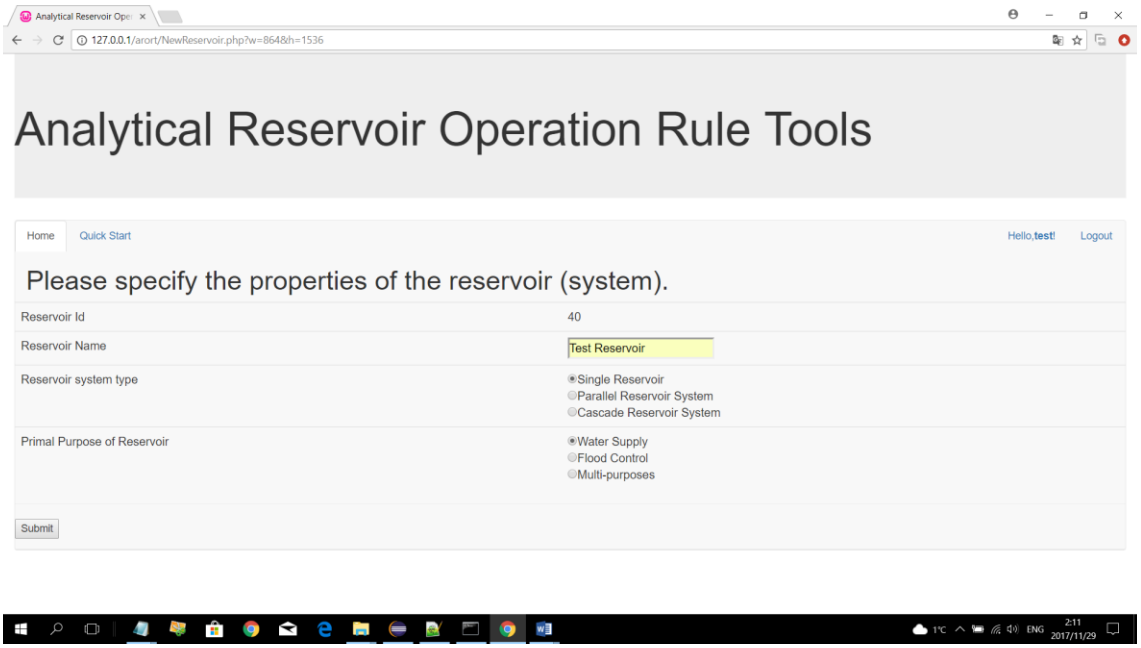


Figure 4.3 Setting up a water supply reservoir

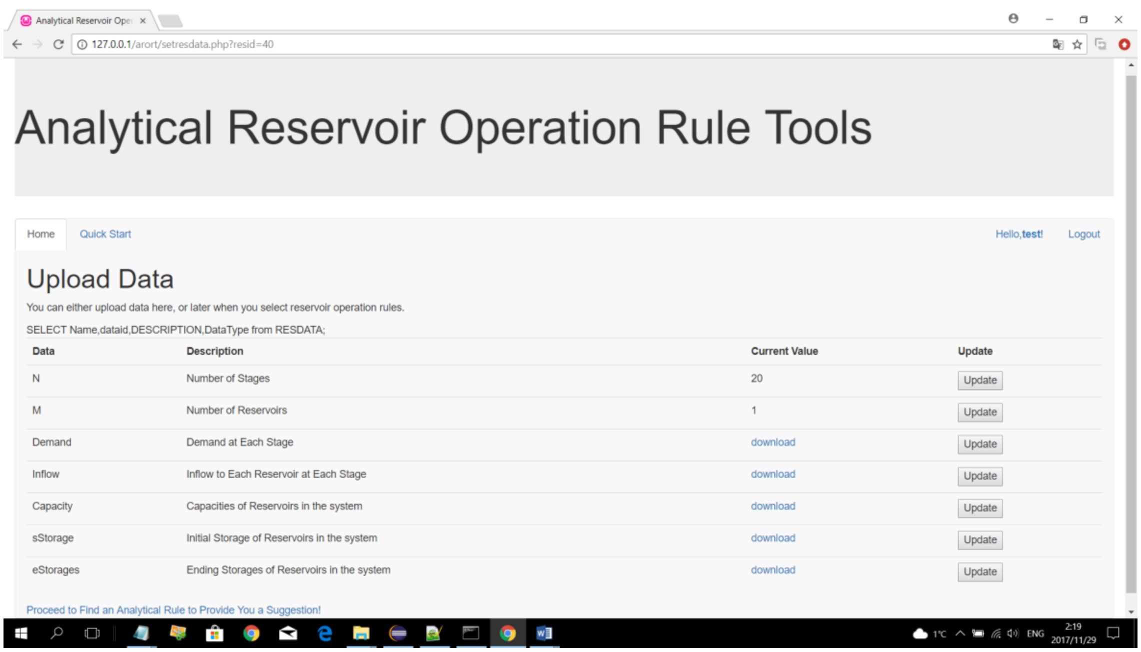


Figure 4.4. Uploading data

After proceeding to the rule list (as shown in Figure 4.5), a proper rule is selected (the rule shown in Figure 4.5 with Rule ID 1), and the result of the rule is visualized as shown in Figure 4.6, which shows the optimal release decision and storage at each stage.

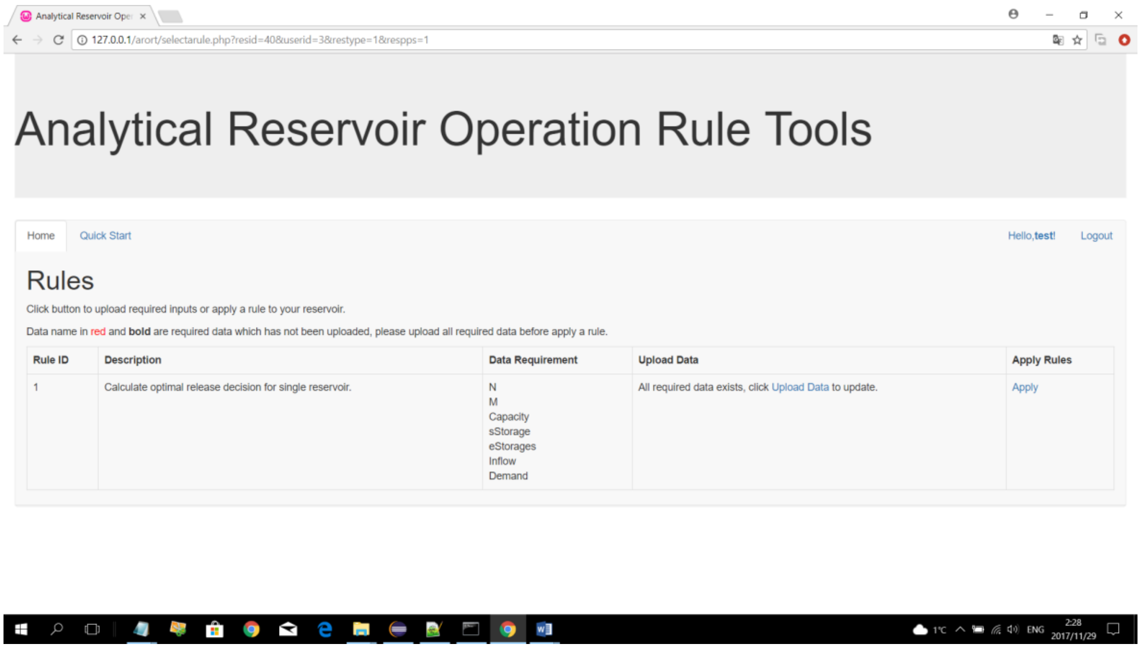


Figure 4.5 Choosing analytical rules available for a single water supply reservoir

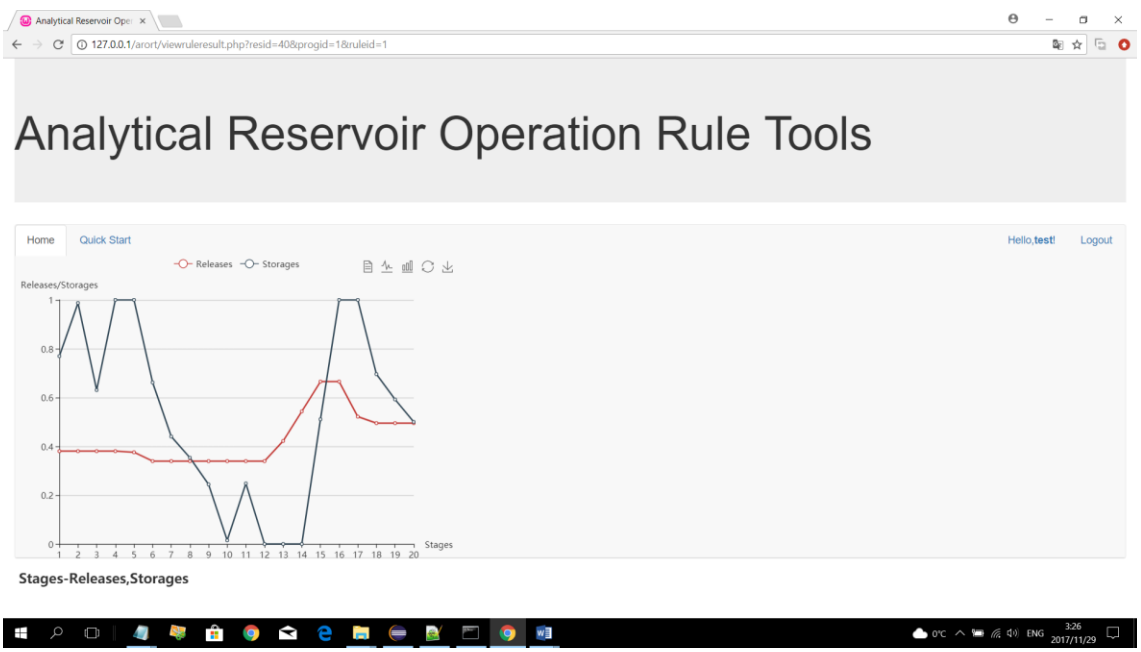


Figure 4.6 Visualizing results

4.3 Summary

In this section, a prototype reservoir operation decision support system based on analytical reservoir operation rules is developed. This system provides a web interface for users to set up reservoirs, upload data, and provide a list of applicable analytical reservoir operation rules based on the inputs from the user. After running the program, the user can view the results.

This prototype system shows the potential of applying analytical reservoir operation rules for practical purposes. Though the analytically derived reservoir operation rules can be limited due to simplifications and assumptions required by the analysis, the derived rules are mostly generic; furthermore, the generic rules can be used for designing computationally effective algorithms. The development of a decision support system provides an implementation of the theoretically derived or empirical rules directly for guiding real-world reservoir operations.

The current system can be used to provide optimal release decisions for single reservoir, optimal release decisions for system of reservoirs in parallel, as well as effective forecast horizons for single reservoir operation under uncertain forecast. The system also provides a flexible framework for additional analytical rules to be added to the system. For an analytical reservoir operation rule that is not included in the system, as long as the program of the rule is developed following some simple requirements on the input and output file formats, the rule can be added to the system by simply uploading the program to the server and updating relevant information in the database. Thus, with the capability of adding more rules, the system can be updated to solve various reservoir operation issues.

CHAPTER 5: CONCLUSIONS

This study applies an analytical approach based on the Karush–Kuhn–Tucker (KKT) conditions, to solve reservoir operation problems, in particular, to establish procedures to determine the effective forecast horizon (EFH) and to develop an algorithm to solve the optimal solution for a system of multiple reservoirs in parallel. In addition, a prototype reservoir operation decision support system is developed to demonstrate the use of the analytical results to practical problems.

In the Chapter 2, a multi-stage optimization model is set up to derive the properties of optimal release decisions for a system of reservoir in parallel with a single demand site, and an algorithm is further developed based on the analytical results to solve for the optimal solution for such a system. Through the analytical derivation, we first identify feasible combinations of the release conditions, storage conditions and the marginal utility changes between two consecutive stages as part of the optimal solution, and further extend the optimality conditions to the entire study horizon to identify the properties of the optimal operations for such reservoir systems. Further, based on the understanding of the properties of the optimal solution, an algorithm is developed to solve parallel reservoir operation problems, with less computational requirement compared to regular numerical approaches. A synthetic case study is conducted and shows the effectiveness and validity of the algorithm. The limitations of this work include the lack of uncertainty considerations and the generality of the algorithm to be applied to different types of reservoir systems. However, this work still provides an example of applying analytical results for algorithm development.

In Chapter 3, to develop procedures to determine the EFH, a multi-stage reservoir operation model is first set up for a single reservoir with single a demand site, and the properties of the optimal solution are analyzed. Based on these properties, the influences of forecast uncertainty on release decisions are discussed, and two special inflow scenarios, i.e., the most optimistic and pessimistic inflow scenario, are identified to provide the upper and lower bounds of the optimal release decision at current stage under forecast with uncertainty. Following this, the concept of the longest forecast horizon (LFH) is proposed as an upper bound for the EFH candidates, and criteria and procedures for determining the LFH and the EFH are developed. Finally, a synthetic case study is conducted, providing insights on the influence of various factors to the EFH and the LFH, while validating the proposed criteria and procedures. Though this work is limited by the assumptions required by the theoretical model, and the simplified representation of the forecast uncertainty, it provides a set of practical criteria and procedures to determine the EFH and the LFH. To further improve the work of this thesis, more analytical work can be done to show the relationship between EFH, LFH and other factors, and a better representation of forecast uncertainty, e.g., uncertainty represented as probability distribution, can also help better understand how the forecast uncertainty affects the optimal release decision.

In Chapter 4, a prototype reservoir operation decision support system is developed based on the analytical results obtained from this thesis and other related studies. Users can use a graphical user interface (GUI) to upload data, execute model, and receive information support for reservoir operation. Though the results might be limited by the assumptions and formulation of the theoretical model, it provides a new approach to optimize reservoir operation activities, which is usually not limited by heavy computational requirements as compared to the numerical approaches used for reservoir operation problems, such as dynamic programming. This prototype system, though still needing further improvement, provides a new perspective on how to apply analytical results for practical purposes.

As a summary, this thesis makes efforts on the derivation and application of analytical results to optimize reservoir operations. In particular, this work focuses on the following three aspects: using analytical results to develop algorithms, i.e., the algorithm for parallel reservoir operation problems, using analytical results to prove some properties of the optimal reservoir operation decisions, i.e., establishing criteria and procedures for determining EFH and LFH, and developing applications of analytical results for real-world reservoir operation, i.e., development of the prototype reservoir operation decision support system. In the future, more work will be needed to explore more possibilities of applying analytical approaches to addressing the various reservoir operation issues and using analytical results for practical purposes.

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APPENDIX A: DERIVATION

The model is formulated as follows:

$$\text{Obj.}\ \ \max \sum_{i=1}^{N} b_i \left(\sum_{j=1}^{M} r_{i,j} \right) \tag{A.1}$$

$$s.t.$$

$$s_{i-1,j} + i_{i,j} - r_{i,j} = s_{i,j}, \qquad for\ i = 1, \dots, N\ and\ j = 1,2, \dots, M \tag{A.2}$$

$$s_{i,j} \geq 0, \qquad for\ i = 1, \dots,\ N-1\ and\ j = 1,2, \dots, M \tag{A.3}$$

$$s_{i,j} \leq K_j, \qquad for\ i = 1, \dots,\ N-1\ and\ j = 1,2, \dots, M \tag{A.4}$$

$$r_{i,j} \geq 0, \qquad for\ i = 1,2, \dots,\ N\ and\ j = 1,2, \dots, M \tag{A.5}$$

Here, i is the index of time and j is the index of reservoirs, M is the total number of reservoirs and *N* is the total number of stages, $b_i(r)$ is the concave utility function of water supply at stage i, $s_{i,j}$ is the storage of reservoir *j* at the end of stage i, the initial and ending storage of reservoir *j* is given as $s_{0,j}$ and $s_{N,j}$ respectively, $r_{i,j}$ is the release of reservoir *j* during stage i, $i_{i,j}$ is the inflow to reservoir *j* during stage i, and K_j is the storage capacity of reservoir *j*.

Transform for applying KKT condition.

$$\text{Obj.}\ \ \max \sum_{i=1}^{N} b_i \left(\sum_{j=1}^{M} r_{i,j} \right) \tag{A.6}$$

$$s.t.$$

$$s_{i-1,j} + i_{i,j} - r_{i,j} - s_{i,j} = 0, \qquad for\ i = 1, \dots, N\ and\ j = 1,2, \dots, M \tag{A.7}$$

$$-s_{i,j} \leq 0, \qquad for\ i = 1, \dots,\ N-1\ and\ j = 1,2, \dots, M \tag{A.8}$$

$$s_{i,j} \leq K_j, \qquad for\ i = 1, \dots,\ N-1\ and\ j = 1,2, \dots, M \tag{A.9}$$

$$-r_{i,j} \leq 0, \qquad for\ i = 1,2, \dots,\ N\ and\ j = 1,2, \dots, M \tag{A.10}$$

Apply KKT condition.

$$-\frac{\partial}{\partial r_{IJ}}\sum_{i=1}^{N} b_i\left(\sum_{j=1}^{M} r_{i,j}\right)+\frac{\partial}{\partial r_{IJ}}\sum_{i=1}^{N}\sum_{j=1}^{M}\lambda_{b,i,j}\left(s_{i,j}-s_{i-1,j}-i_{i,j}+r_{i,j}\right)$$
$$+\frac{\partial}{\partial r_{IJ}}\sum_{i=1}^{N-1}\sum_{j=1}^{M}\lambda_{e,i,j}\left(-s_{i,j}\right)+\frac{\partial}{\partial r_{IJ}}\sum_{i=1}^{N-1}\sum_{j=1}^{M}\lambda_{f,i,j}\left(s_{i,j}-K_j\right)$$
$$+\frac{\partial}{\partial r_{IJ}}\sum_{i=1}^{N}\sum_{j=1}^{M}\lambda_{r,i,j}\left(-r_{i,j}\right)=0,$$
$$for\ I=1,2,\dots,N,\ J=1,2,\dots,M \qquad (A.11)$$

$$-\frac{\partial}{\partial s_{IJ}}\sum_{i=1}^{N} b_i\left(\sum_{j=1}^{M} r_{i,j}\right)+\frac{\partial}{\partial s_{IJ}}\sum_{i=1}^{N}\sum_{j=1}^{M}\lambda_{b,i,j}\left(s_{i,j}-s_{i-1,j}-i_{i,j}+r_{i,j}\right)$$
$$+\frac{\partial}{\partial s_{IJ}}\sum_{i=1}^{N-1}\sum_{j=1}^{M}\lambda_{e,i,j}\left(-s_{i}\right)+\frac{\partial}{\partial s_{IJ}}\sum_{i=1}^{N-1}\sum_{j=1}^{M}\lambda_{f,i,j}\left(s_{i}-K_j\right)$$
$$+\frac{\partial}{\partial s_{IJ}}\sum_{i=1}^{N}\sum_{j=1}^{M}\lambda_{r,i,j}\left(-r_{i,j}\right)=0,$$
$$for\ I=1,\dots,N-1\ and\ J=1,2,\dots,M \qquad (A.12)$$

$$-s_{i,j}\le 0,\qquad for\ i=1,\dots,N-1\ and\ j=1,2,\dots,M \qquad (A.13)$$

$$s_{i,j}\le K_i,\qquad for\ i=1,\dots,N-1\ and\ j=1,2,\dots,M \qquad (A.14)$$

$$\lambda_{e,i,j}s_{i,j}=0,\qquad for\ i=1,\dots,N-1\ and\ j=1,2,\dots,M \qquad (A.15)$$

$$\lambda_{f,i,j}\left(s_{i,j}-K_i\right)=0,\qquad for\ i=1,\dots,N-1\ and\ j=1,2,\dots,M \qquad (A.16)$$

$$\lambda_{r,i,j}r_{i,j}=0,\qquad for\ i=1,\dots,N\ and\ j=1,2,\dots,M \qquad (A.17)$$

$$\lambda_{e,i,j}\ge 0,\qquad for\ i=1,\dots,N-1\ and\ j=1,2,\dots,M \qquad (A.18)$$

$$\lambda_{f,i,j}\ge 0,\qquad for\ i=1,\dots,N-1\ and\ j=1,2,\dots,M \qquad (A.19)$$

$$\lambda_{r,i,j}\ge 0,\qquad for\ i=1,2,\dots,N\ and\ j=1,2,\dots,M \qquad (A.20)$$

Here, $\lambda_{b,i,j}$ is the shadow price for the mass balance constrain, Eq. (A.2), of reservoir *j* at stage *i*, $\lambda_{e,i,j}$ is the shadow price for non-negative storage constraints, Eq. (A.3), of reservoir *j* at the end of stage *i*, $\lambda_{f,i,j}$ is the shadow price for storage capacity constraints, Eq. (A.4), of reservoir *j* at the end of stage *i*, and $\lambda_{r,i,j}$ is the shadow price for non-negative release constraints, Eq. (A.5), of reservoir j on stage i. Eq. (A.21) – Eq. (A.22) could be obtained by simplifying Eq. (A.12) – Eq. (A.20).

$$\lambda_{b,i,j} = b_i'\left(\sum_{j=1}^{M} r_{i,j}\right) + \lambda_{r,i,j}, \qquad for\ i = 1,2,\dots,N\ and\ j = 1,2,\dots,M \qquad (A.21)$$

$$\lambda_{b,i+1,j} + \lambda_{e,i,j} - \lambda_{f,i,j} = \lambda_{b,i,j}, \qquad for\ i = 1,2,\dots,N-1\ and\ j = 1,2,\dots,M \quad (A.22)$$

Substitute Eq. (A.21) to Eq. (A.22),

$$b_{i+1}'\left(\sum_{j=1}^{M} r_{i+1,j}\right) + \lambda_{r,i+1,j} + \lambda_{e,i,j} - \lambda_{e,i,j} = b_i'\left(\sum_{j=1}^{M} r_{i,j}\right) + \lambda_{r,i,j},$$
$$for\ i = 1,2,\dots,N-1\ \ and\ j = 1,2,\dots,M \qquad (A.23)$$

$$b_{i+1}'\left(\sum_{j=1}^{M} r_{i+1,j}\right) + \lambda_{e,i,j} - \lambda_{f,i,j} = b_i'\left(\sum_{j=1}^{M} r_{i,j}\right) + \left(\lambda_{r,i,j} - \lambda_{r,i+1,j}\right),$$
$$for\ i = 1,2,\dots,N-1\ and\ j = 1,2,\dots,M \qquad (A.24)$$

APPENDIX B: PROOF

If we solve the optimization problem under the assumption (Assumption I) that all reservoirs will release as much water as possible before a MU decrease without considering future droughts, i.e., all reservoirs storages achieve the lowest possible value when MU decreases, we will obtain a result more conservative for current stages. Thus, as shown in Figure B.1, in the optimal solution solved under Assumption I, there is no water saved from earlier stages with high MU, e.g., stages of Piece 1 and Piece 2, to later stages with high MU, e.g., stages of Piece 4, while in the optimal solution of the original problem that is not subject to Assumption I, some water can might be saved from earlier stages with high MU to later stages with high MU. To obtain the optimal solution of the original problem from the optimal solution solved under Assumption I, instead of releasing all available water before MU decreases, e.g., at stages of Piece 1 and Piece 2, some reservoirs might not be empty when MU decreases and some water might be saved to later stages with high MU, e.g., stages of Piece 4, if the MU is higher at later stages than that of the earlier stages. The maximum amount of water saved is constrained by the excessive storage capacity of the reservoirs after storing as much water as possible during the stages with low MU, e.g. stages of Piece 3 in Figure B.1.

We will focus on the first occurrence of MU increase in the optimal solution solved under Assumption I. Under the following four situations, the relationship between the optimal solution solved for the original problem that is not subject to Assumption I and the optimal solution solved under Assumption I are discussed.

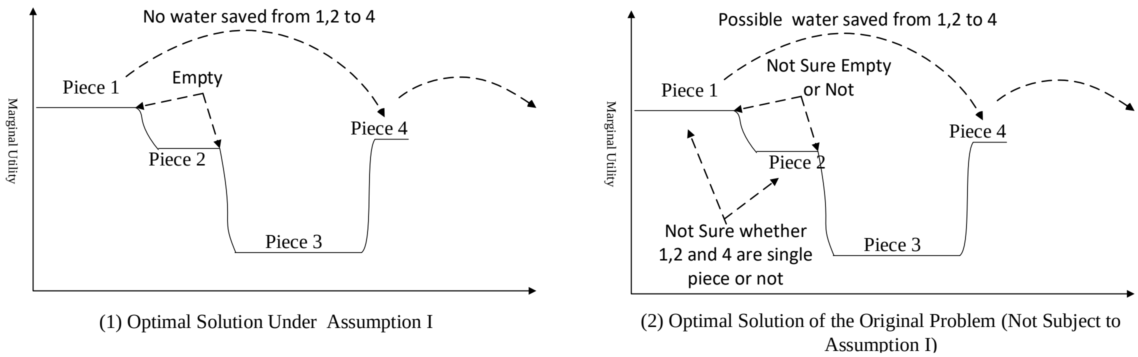

Figure B.1 Illustration of the optimal solutions solved with/without Assumption I

(1) Assume all reservoirs become full at the first occurrence of MU increase in the study horizon with the optimal solution solved under Assumption I. By removing Assumption I from the optimal solution solved under Assumption I, no more water can be saved from stages of Piece 1 & 2 to stages of Piece 4, as the storages are already full at the end of Piece 3. It is possible that there will be water saved from Piece 4 to the future, which will possibly make the MU of Piece 4 higher, and thus, MU of Piece 4 is still higher than that of Piece 3, which means reservoirs will still be full at the end of Piece 3 after removing Assumption I. Thus, if all reservoirs become full at the first occurrence of MU increase in the study horizon with the optimal solution solved with Assumption I, the optimal release decision before the reservoirs are full solved with Assumption I will not be subject to Assumption I.

(2) Assume that in the optimal decisions solved with Assumption I, there is no decrease of MU before the first occurrence of MU increase (i.e., there is a single MU before the first occurrence of the MU increase) (Piece 1 & 2 in Figure B.1 not exist). By removing Assumption I from the optimal solution solved under Assumption I, there might be more water saved from Piece 4 to the future, which will possibly make the MU of Piece 4 higher, and thus, MU of Piece 4 is still higher than that of Piece 3, thus, reservoirs should save as much water as possible during stages of Piece 3, which is the same as the optimal solution

solved under Assumption I. Thus, if in the optimal decisions solved with Assumption I, there is no decrease of MU before the first occurrence of MU increase (i.e., there is a single MU before the first occurrence of the MU increase), then, the optimal release decision before the first occurrence of MU increase will be the same as the optimal release decision of the original problem that is not subject to Assumption I.

(3) Assume that no MU increase occurs in the study horizon in the optimal release decision solved with Assumption I (Piece 4 in Figure B.1 does not exist). Therefore, the MU keeps decreasing within the study horizon, thus, by removing Assumption I from the optimal solution solved under Assumption I, there is no need to save water from any piece to the future, as the MU at any stage is higher than all future stages in the study horizon. Thus, if no states with increasing MU occurs in the optimal release decision solved with Assumption I, then the optimal release decision will be the same as the optimal decision of the original problem that is not subject to Assumption I.

(4) Assume, none of (1), (2) or (3) is satisfied in the optimal release decision under Assumption I. By removing Assumption I from the optimal solution solved under Assumption I, it is possible that there is some water saved from Piece 1 & 2 to the future after Piece 3, e.g., Piece 4. After this reallocation, the MU of Piece 1, 2 and Piece 4 will still be greater or equal to the lowest MU of these three pieces under the optimal solution solved under Assumption I, and thus, is still greater than the MU of Piece 3. The reason is given as follows. The MUs of Piece 1 & 2 have already reached a minimum possible value in the optimal solution solved under Assumption I, as all reservoir storages are the lowest possible storage when MU decreases, thus, the MU of Piece 1 & 2 cannot be lower than that of Piece 3 by removing Assumption I. After removing Assumption I, if the MU of Piece 4 becomes lower by receiving some water from Piece 1 & 2, then, it cannot be lower than the lower one of the MUs of Piece 1 & 2 to achieve optimality, thus, is still higher than that of Piece 3. For reservoirs not full at the end of Piece 3 and not releasing on Piece

3 under the optimal solution solved with Assumption I, it is possible for them to save water from stages of Piece 1 & 2 to stages after Piece 3 under the optimal decision of the original problem that is not subject to Assumption I, thus, might be not empty at the end of Piece 2. However, there is still no release from such reservoirs during stages of Piece 3, as the reservoir is not empty when the MU decreases at the end of Piece 2. Thus, after removing Assumption I, the release decision of Piece 3 will not be changed, though the storage of the reservoirs during stages of Piece 3 might be different. For other reservoirs, under the optimal release decision solved with Assumption I, they are already full at the end of Piece 3, thus, it is not possible for them to be not empty at the end of Piece 2 to save water for the future after removing Assumption I. Thus, if neither the condition of (1), (2) nor (3) are satisfied in the optimal release decision under Assumption I, then the optimal release decision between the first occurrence of the MU increase and the adjacent previous occurrence of the decrease of the MU will be the same as the optimal release decision of the original problem that is not subject to Assumption I.

APPENDIX C: DERIVATION

Formulation:

$$\text{Obj. } \max \sum b_i(r_i) \tag{C.1}$$

$$s.t.$$

$$s_{i-1} + I_i - r_i = s_i, \qquad for\ i = 1,2,\dots,N \tag{C.2}$$

$$s_i \geq 0, \qquad for\ i = 1,2,\dots,N-1 \tag{C.3}$$

$$s_i \leq K, \qquad for\ i = 1,2,\dots,N-1 \tag{C.4}$$

where $b_i(r_i)$ is the concave utility function for stage i, s_i is the storge of reservoir at the end of stage i, I_i is the inflow forecast during stage i, r_i is the release during stage i, K is the capacity of the reservoir and N is the attempted forecast horizon.

Using KKT condition, the optimization problem could be transformed as follows.

$$\text{Obj. } \min -\sum b_i(r_i) \tag{C.5}$$

$$s.t.$$

$$s_{i-1} + i_i - r_i = s_i, \qquad for\ i = 1,2,\dots,N \tag{C.6}$$

$$-s_i \leq 0, \qquad for\ i = 1,\dots,N-1 \tag{C.7}$$

$$s_i \leq K, \qquad for\ i = 1,\dots,N-1 \tag{C.8}$$

The necessary condition for optimal solution is given as follows.

$$-\frac{\partial}{\partial r_I}\sum_{i=1}^{N} b_i(r_i) + \sum_{i=1}^{N} \lambda_{b,i}\frac{\partial}{\partial r_I}(s_i - s_{i-1} - i_i + r_i) + \sum_{i=1}^{N-1} \lambda_{e,i}\frac{\partial}{\partial r_I}(-s_i)$$
$$+ \sum_{i=1}^{N-1} \lambda_{f,i}\frac{\partial}{\partial r_I}(s_i - K) = 0, \qquad for\ I = 1,2,\dots,N \tag{C.9}$$

$$-\frac{\partial}{\partial s_I}\sum_{i=1}^{N} b_i(r_i) + \sum_{i=1}^{N} \lambda_{b,i}\frac{\partial}{\partial s_I}(s_i - s_{i-1} - i_i + r_i) + \sum_{i=1}^{N-1} \lambda_{e,i}\frac{\partial}{\partial s_I}(-s_i)$$

$$+\sum_{i=1}^{N-1} \lambda_{f,i}\frac{\partial}{\partial s_I}(s_i - K) = 0, \qquad for\ I = 1, \dots, N-1 \qquad (C.10)$$

$$-s_i \le 0, \qquad for\ i = 1, \dots, N-1 \qquad (C.11)$$

$$s_i \le K, \qquad for\ i = 1, \dots, N-1 \qquad (C.12)$$

$$\lambda_{e,i} s_i = 0, \qquad for\ i = 1, \dots, N-1 \qquad (C.13)$$

$$\lambda_{f,i}(s_i - K) = 0, \qquad for\ i = 1, \dots, N-1 \qquad (C.14)$$

$$\lambda_{e,i} \ge 0, \qquad for\ i = 1, \dots, N-1 \qquad (C.15)$$

$$\lambda_{f,i} \ge 0, \qquad for\ i = 1, \dots, N-1 \qquad (C.16)$$

where $\lambda_{e,i}$ is the shadow price for non-negative storage constraints at stage *i*, $\lambda_{f,i}$ is the shadow price for the capacity constraint at stage *i* and $\lambda_{b,i}$ is the shadow price for the mass balance constraint at stage *i*.

From Eq. (C.9) we can have

$$-\frac{\partial b_i(r_i)}{\partial r_i} + \lambda_{b,i} = 0, \qquad for\ i = 1,2, \dots, N \qquad (C.17)$$

From Eq. (C.10) we can have

$$\lambda_{b,i-1} - \lambda_{b,i} - \lambda_{e,i} + \lambda_{f,i} = 0, \qquad for\ i = 1, \dots, N-1 \quad (C.18)$$

APPENDIX D: PROOF

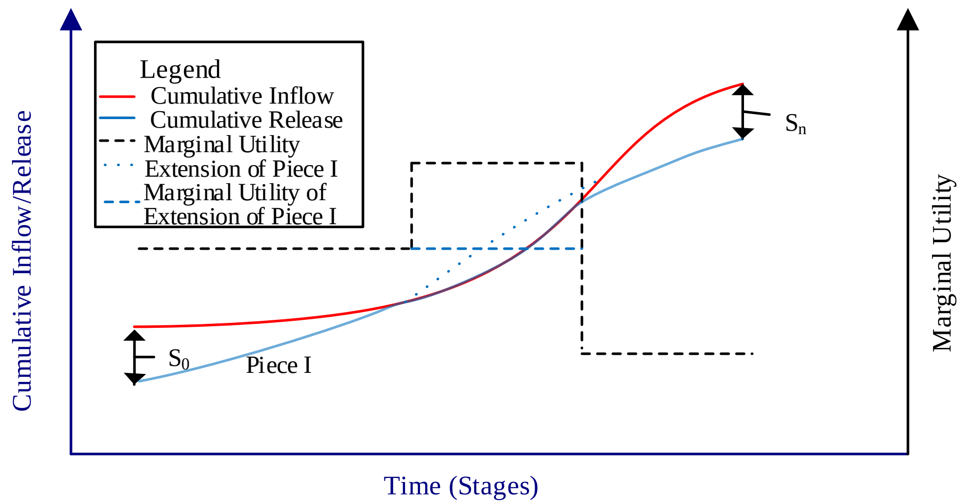

Figure D.1 Illustration of MU changes under optimal release decisions

First, we show that the cumulative release curve will be tangent to the cumulative inflow curve when an empty state occurs. As shown in the Figure D.1, if the intersection of these two curves are not tangent to each other, i.e., the intersection point is a cross-over point. Then, if by extending Piece I of the cumulative delivery curve with identical MU (the extended part is shown as dotted blue line), there is a time period, during which the extended delivery curve with the same MU as Piece I is above the actual cumulative inflow curve, and thus, is above the actual cumulative water delivery curve. This means during this period, the actual MU is higher than the MU of Piece I. Then this contradicts with the relationship derived for stages with non-negative storage constraint binding, i.e.,

$$b_i'(r_i^*) = b_{i+1}'(r_{i+1}^*) + \lambda_{e,i} \qquad (D.1)$$

Thus, if the point is a point of cross-over, this relationship shown in Eq. (D.1) will be violated.

Same derivation can be done for a stage with binding storage capacity constraint. It could be shown that if the intersection of these two curves are not tangent to each other, namely, the intersection point is a cross-over point. Then, there is a time period, during which the actual MU is lower than the MU of the periods before the stage with binding storage capacity constraint. Then this contradicts with the relationship derived for stages with storage capacity constraint binding, i.e.,

$$b'_{i-1}(r^*_{i-1}) = b'_i(r^*_i) - \lambda_{f,i} \qquad (D.2)$$

Thus, if the point is a point of cross-over, this relationship shown in Eq. (D.2) will be violated.

APPENDIX E: PROOF

Under Case 4 (the non-negative storage constraint becomes binding for the first time after the initial storage, and then the storage capacity constraint becomes binding at a later stage), two scenarios, i.e. future water abundant scenario and future water stress scenario, are discussed.

Figure E.1 shows future water abundant scenario, when more water should be released in T_f. As shown in the figure, even if more water will be released during T_f, only release beyond T_d will be increased. The reservoir is empty at T_d ,and thus, the release before T_d has already reached a maximum. Otherwise, if the release within T_d is increased, the cumulative water delivery curve will no longer be tangent with the cumulative inflow curve at T_d, which contradicts with the optimal solution solved from the optimization model. Therefore, even if there is more inflow in the future beyond T_f, the decision in T_d will not be changed.

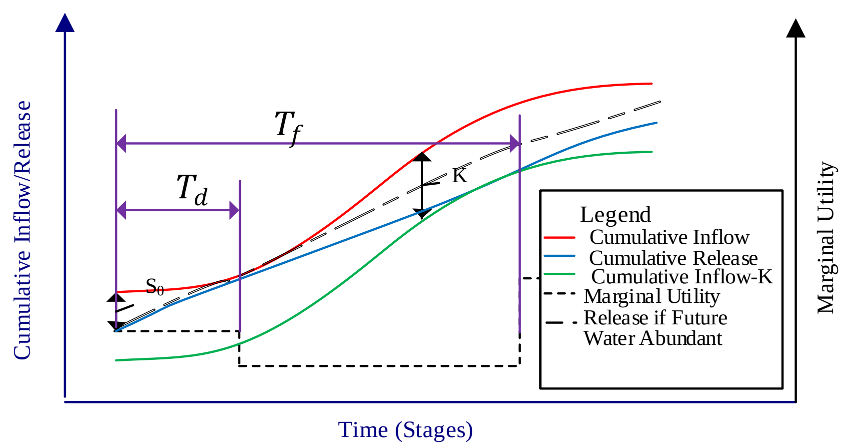

Figure E.1 Water Abundant Future

The other scenario is drought in the future. For this situation, as the storage at T_f has already reached the maximum, no more water can be saved before T_f. As shown in Figure

E.2, if reduce release before T_f, the cumulative water delivery will decrease at T_f, and thus the cumulative water delivery curve and the cumulative inflow curve moved downward by the storage capacity will cross over with each other, and thus, is not the optimal solution. Therefore, with drought in the future, the optimal release decision within T_d remains unchanged.

Thus, regardless of the inflow condition beyond T_f, the optimal decision will not be changed in the T_d given information in T_f.

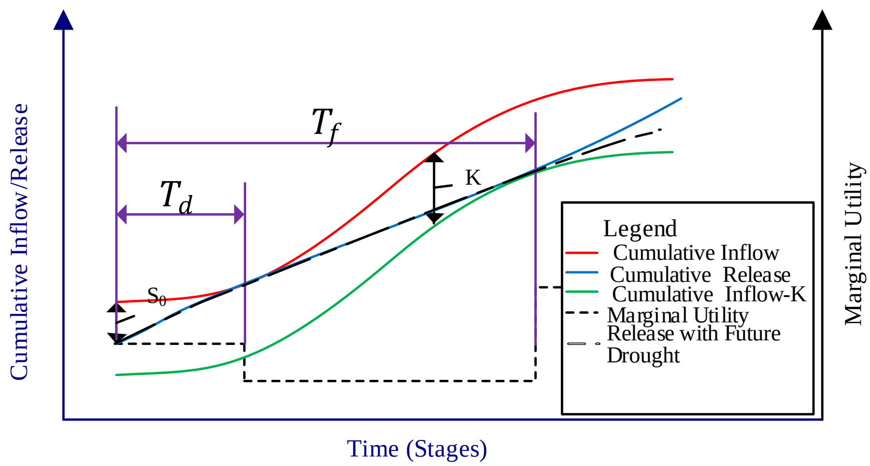

Figure E.2 Water Stress Future

Similar analysis can be conducted for Case 5 (the storage capacity constraint becomes binding for the first time after the initial storage, and then the non-negative storage constraint becomes binding at a later stage).

APPENDIX F: PROOF

In this appendix, the following statement is proved. For a given inflow forecast for future N stages, $I_1, I_2, \ldots, I_N$, and corresponding inflow uncertainty under certain confidence levels, such as $[\underline{I_1}, \overline{I_1}]$, $[\underline{I_2}, \overline{I_2}]$, … , $[\underline{I_N}, \overline{I_N}]$, and an estimation of ending storage, $\left[\underline{S_N}, \overline{S_N}\right]$, among all possible inflow scenarios, the most optimistic scenario, i.e., $I_{1,opt}, I_{2,opt}, \ldots, I_{N,opt}$, which has the highest cumulative inflow at all stages over all possible scenarios, together with the lower bound of ending storage, $\underline{S_N}$, provides the largest amount of release, $\bar{R}_1^*$, at current stage, and the most pessimistic scenario, i.e., $I_{1,pes}, I_{2,pes}, \ldots, I_{N,pes}$, which has the lowest cumulative inflow at all stages over all possible scenarios, together with the upper bound of ending storage, $\overline{S_N}$, provides the smallest amount of release, $\underline{R_1^*}$, at current stage.

Proof by contradiction is applied here.

Assume that the optimal release decision, R_1', solved under inflow forecast, $I_1', I_2', \ldots, I_N'$, which is different from the most optimistic one, is greater than the optimal release decision solved under most optimistic forecast, $\bar{R}_1^*$, i.e.,

$$\sum_{i=1}^{t} I_i' \leq \sum_{i=1}^{t} I_{i,opt}, \quad \forall t = 1,2, \ldots, N \tag{F.1}$$

$$R_1' > \bar{R}_1^* \tag{F.2}$$

thus, over the study horizon, N,

$$\sum_{i=1}^{N} I_i' \leq \sum_{i=1}^{N} I_{i,opt} \tag{F.3}$$

and according to the characteristics of utility function $b_i(r_i)$,

$$b_1(R_1') > b_1(\bar{R}_1^*) \tag{F.4}$$

$$b_1'(R_1') < b_1'(\bar{R}_1^*) \tag{F.5}$$

There should exists at least one stage *k* (and assume *k* is the first one) satisfying Eq. (F.6) & Eq. (F.7),

$$R_k' > \bar{R}_k^* \tag{F.6}$$

$$R_{k+1}' < \bar{R}_{k+1}^* \tag{F.7}$$

where $\bar{R}_k^*$ is the optimal release decision at stage *k* solved under the most optimistic forecast, and R_k' is the optimal release decision at stage *k* solved under inflow forecast $I_1', I_2', \dots, I_N'$. Then,

$$b_k'(R_k') < b_k'(\bar{R}_k^*) \tag{F.8}$$

$$b_{k+1}'(R_{k+1}') > b_{k+1}'(\bar{R}_{k+1}^*) \tag{F.9}$$

otherwise,

$$\sum_{i=1}^{N} R_i' > \sum_{i=1}^{N} \bar{R}_i^* \tag{F.10}$$

$$\sum_{i=1}^{N} R_i' + S_N - S_0 > \sum_{i=1}^{N} \bar{R}_i^* + \underline{S_N} - S_0 \tag{F.11}$$

$$\sum_{i=1}^{N} I_i' > \sum_{i=1}^{N} I_{i,opt} \tag{F.12}$$

which will contradict with Eq. (F.3).

According to Eq. (F.8) & (F.9), there should be following several possibilities. In the following discussion, for simplicity, the most optimistic scenario will be referred as Case 1 and the other inflow scenario will be referred as Case 2, and $\bar{S}_i, I_{i,opt}, \bar{R}_i^*$ will be used to represent storage, inflow and release for Case 1, and S_i', I_i', R_i' will be used to represent storage, inflow and release for Case 2.

(1) $b_k'(R_k') = b_{k+1}'(R_{k+1}')$ and $b_k'(\bar{R}_k^*) > b_{k+1}'(\bar{R}_{k+1}^*)$. Therefore, for Case 1, according to the analytical solution, there is an empty state between stage *k* and stage *k*+1. Thus,

$$S'_{k+1} \geq \bar{S}_{k+1} = 0 \quad (F.13)$$

combining the assumption shown by Eq. (F.1),

$$\sum_{i=1}^{k} I'_i + S_0 - S'_{k+1} \leq \sum_{i=1}^{k} I_{i,opt} + S_0 - \bar{S}_{k+1} \quad (F.14)$$

$$\sum_{i=1}^{k} R'_i \leq \sum_{i=1}^{k} \bar{R}_i \quad (F.15)$$

This contradicts with the assumption that k is the first stage satisfying Eq. (F.6) and Eq. (F.7).

(2) $b'_k(R'_k) < b'_{k+1}(R'_{k+1})$ and $b'_k(\bar{R}^*_k) = b'_{k+1}(\bar{R}^*_{k+1})$. Therefore, for Case 2, according to the analytical solution, there is a full state between stage k and stage k+1. Thus,

$$K = S'_{k+1} \geq \bar{S}_{k+1} \quad (F.16)$$

which is the same as Eq. (F.13) and contradicts with the assumption that k is the first stage satisfying Eq. (F.6) and Eq. (F.7).

(3) $b'_k(R'_k) < b'_{k+1}(R'_{k+1})$ and $b'_k(\bar{R}^*_k) > b'_{k+1}(\bar{R}^*_{k+1})$. Therefore, for Case 1, according to the analytical solution, there is an empty state between stage k and stage k+1, and for Case 2, there is a full state between stage k and stage k+1. Thus,

$$K = S'_{k+1} \geq \bar{S}_{k+1} = 0 \quad (F.17)$$

which is also the same as Eq. (F.13) and contradicts with the assumption that k is the first stage satisfying Eq. (F.6) and Eq. (F.7).

(4) $b'_k(R'_k) < b'_{k+1}(R'_{k+1})$ and $b'_k(\bar{R}^*_k) < b'_{k+1}(\bar{R}^*_{k+1})$. Therefore, for both Case 1 and Case 2, according to the analytical solution, there is a full state between stage k and stage k+1. Thus,

$$K = S'_{k+1} = \bar{S}_{k+1} = K \quad (F.18)$$

combining the assumption shown by Eq. (F.1),

$$\sum_{i=1}^{k} I'_i + S_0 - S'_{k+1} \leq \sum_{i=1}^{k} \bar{I}_i + S_0 - \bar{S}_{k+1} \quad (F.19)$$

which is the same as Eq. (F.14) and contradicts with the assumption that k is the first stage satisfying Eq. (F.6) and Eq. (F.7).

(5) $b'_k(R'_k) > b'_{k+1}(R'_{k+1})$ and $b'_k(\bar{R}^*_k) > b'_{k+1}(\bar{R}^*_{k+1})$. Therefore, for both Case 1 and Case 2, according to the analytical solution, there is an empty stage between stage k and stage k+1. Thus,

$$0 = S'_{k+1} = \bar{S}_{k+1} = 0 \qquad (F.20)$$

which is also the same as Eq. (F.18) and is contradicting with the assumption that k is the first stage satisfying Eq. (F.6) and Eq. (F.7).

Therefore, it is shown that among all possible inflow scenarios, the most optimistic scenario, i.e., $\overline{I_1}, \overline{I_2}, \dots, \overline{I_N}$, will provide the largest amount of release, $\bar{R}^*_1$, at current stage.

Similarly, it can be proved that Eq. (F.21) contradicts with Eq. (F.22) under all possibilities.

$$\sum_{i=1}^{t} I'_i \geq \sum_{i=1}^{t} I_{i,pes}, \quad \forall t = 1,2,\dots,N \qquad (F.21)$$

$$R'_1 < \bar{R}^*_1 \qquad (F.22)$$

Thus, among all possible inflow scenarios, the most pessimistic scenario provides the lowest amount of release, $\underline{R}^*_1$, at current stage.

APPENDIX G: PROOF

The following property is used when searching the longest forecast horizon (LFH). For any inflow scenario, if the release decisions at the current stage solved with a full or empty ending storage as constraints are identical, then the release decision at the current stage should be irrelevant to the ending storage.

This is a special case of the statement proved in Appendix F. For a given deterministic inflow scenario (i.e., only one possible inflow scenario) and an ending storage ranging from empty to full, according to Appendix D, the lower bound of the optimal release decision at the current stage is solved under the given deterministic inflow with full ending storage as constraint and the upper bound of the optimal release decision at the current stage is solved under the given deterministic inflow with empty ending storage as constraint. For a given deterministic inflow if the lower bound and upper bound of the optimal release decision at current stage are the identical, then, a consistent optimal release decision will be solved under any ending storage constraint. Therefore, if for each of the most optimistic and most pessimistic inflow forecast, the release decision is identical with empty or full ending storage as constraints, then the forecast length satisfies the requirement that the release decision error bound at current stage is irrelevant to the ending storage at the end of forecast horizon.